

# Using Pooled Information and Bootstrap Methods to Assess Debt Sustainability in Low Income Countries

*Constantino Hevia*

The World Bank  
Development Research Group  
Macroeconomics and Growth Team  
February 2012



## Abstract

Conventional assessments of debt sustainability in low income countries are hampered by poor data and weaknesses in methodology. In particular, the standard International Monetary Fund-World bank debt sustainability framework relies on questionable empirical assumptions: its baseline projections ignore statistical uncertainty, and its stress tests, which are performed as robustness checks, lack a clear economic interpretation and ignore the interdependence between the relevant macroeconomic variables. This paper proposes to alleviate these problems by pooling data from many countries and estimating the shocks and macroeconomic

interdependence faced by a generic, low income country. The paper estimates a panel vector autoregression to trace the evolution of the determinants of debt, and performs simulations to calculate statistics on external debt for individual countries. The methodology allows for the value of the determinants of debt to differ across countries in the long run, and for additional heterogeneity through country-specific exogenous variables. Results in this paper suggest that ignoring the uncertainty and interdependence of macroeconomic variables leads to biases in projected debt trajectories, and consequently, the assessment of debt sustainability.

---

This paper is a product of the Macroeconomics and Growth Team, Development Research Group. It is part of a larger effort by the World Bank to provide open access to its research and make a contribution to development policy discussions around the world. Policy Research Working Papers are also posted on the Web at <http://econ.worldbank.org>. The author may be contacted at [chevia@worldbank.org](mailto:chevia@worldbank.org).

*The Policy Research Working Paper Series disseminates the findings of work in progress to encourage the exchange of ideas about development issues. An objective of the series is to get the findings out quickly, even if the presentations are less than fully polished. The papers carry the names of the authors and should be cited accordingly. The findings, interpretations, and conclusions expressed in this paper are entirely those of the authors. They do not necessarily represent the views of the International Bank for Reconstruction and Development/World Bank and its affiliated organizations, or those of the Executive Directors of the World Bank or the governments they represent.*

# Using Pooled Information and Bootstrap Methods to Assess Debt Sustainability in Low Income Countries

Constantino Hevia\*

JEL codes: F34, F47, H63, H68

Keywords: Low income countries, debt sustainability, panel vector autoregression, bootstrap

Sector board: EPOL

---

\*The author is with the World Bank, DECMG. I am grateful to Aart Kray, Norman Loayza, Claudio Raddatz, and Luis Servén for valuable comments and suggestions. I owe special thanks to Luca Bandiera, Leonardo Hernández, and Juan Pradelli for advice and endless discussions about the current IMF-World Bank Debt Sustainability Framework, and to the World Bank's PREMED for support. I am also thankful to Bledi Celiku for very able research assistance. The findings, interpretations, and conclusions expressed in this paper are entirely those of the author. They do not necessarily reflect the view of the World Bank, those of the Executive Directors, or the governments they represent.

# 1 Introduction

Assessing the sustainability of external debt requires making assumptions about the evolution of its determinants. To make sensible predictions about future debt trajectories, projections need to take into account the uncertainty, co-movement, and feedback effects between the relevant macroeconomic variables. For example, crisis episodes with large increases of the debt burden are typically associated with current account reversals, real exchange rate depreciations, and so forth. It is important to take into account these correlations to provide reasonable projections and, therefore, to assess the sustainability of external debt.

The current joint International Monetary Fund (IMF)–World Bank debt sustainability framework (DSF) for low income countries is based on three pillars.<sup>1</sup> The first pillar is a set of bounds on external debt as a fraction of gross domestic product (GDP) or exports that vary according to the quality of the country’s institutions and policies. The second pillar is a set of projections of key macroeconomic variables coupled with a debt accumulation equation to produce projections of external debt. Countries with current or projected debt indicators above the proposed bounds are considered to have unsustainable levels of debt. The third pillar is a set of “stress tests” designed to check the robustness of the baseline projection. Even if the projected debt level is below the proposed bounds, it might become unsustainable should the country suffer sufficiently large negative shocks. The DSF considers a number of negative shocks on the baseline projections to assess whether current levels of debt are “sufficiently” safe.

The DSF for low income countries, however, has a number of drawbacks.<sup>2</sup> The baseline projection is based on experts’ opinions or on the point estimates of the projections compiled in the IMF’s World Economic Outlook. The framework, however, ignores any uncertainty associated with these forecasts. This is particularly important given the long projection horizon (of 20 years) included in the current framework. Moreover, while the stress tests are

---

<sup>1</sup>The current DSF is described in World Bank and IMF (2009) and Painchaud and Stučka (2011).

<sup>2</sup>Some of these problems were already noted in IMF (2004) and Hostland (2011).

thought to be a way to cope with the uncertainty involved in the baseline projections, their implementation is problematic for, at least, two reasons. First, the framework ignores the co-movements between the macroeconomic variables that determine the evolution of external debt. For example, a negative shock to GDP growth is assumed to have no impact on other macroeconomic variables, like the real exchange rate, the current account, or the interest rate. These variables are kept constant at their baseline projections. Moreover, shocks in the current framework do not have a meaningful economic interpretation. Where do shocks come from? What is their structural interpretation? Second, some argue that the stress tests are too stringent because they consider highly unlikely and pessimistic scenarios. For example, one of the stress tests in the current framework involves a simultaneous worsening in four determinants of debt for two years.<sup>3</sup> There is no rationale for the duration or the size of the proposed shock.

This paper proposes to alleviate these problems by pooling data from many countries and estimating the shocks and co-movements faced by a generic low income country. The methodology is an extension to that proposed by Garcia and Rigobon (2005). I first derive a debt accumulation equation and identify key variables that determine the evolution of foreign debt.<sup>4</sup> Next, I estimate a panel vector autoregression with common slope coefficients and covariance matrix to model the evolution of the determinants of debt. Finally, I perform simulations to generate a large number of future debt trajectories for a given country and compute several statistics on external debt—median forecasts, confidence regions, the probability of debt crossing some threshold, and so forth. These statistics can be used to assess whether external debt is, or is likely to be, “too large” relative to a predetermined criterion.

To perform the simulations, Garcia and Rigobon (2005) use a Monte Carlo method and assume that shocks are drawn from a normal distribution with mean zero and covariance matrix equal to that of the fitted residuals. One contribution of this paper is to replace the

---

<sup>3</sup>This is a combined negative shock to real GDP growth, exports growth, the GDP deflator measured in U.S. dollars, and private transfers and foreign direct investment.

<sup>4</sup>The terms external debt and foreign debt are used interchangeably throughout the paper.

Monte Carlo step with a bootstrap approach. The bootstrap is a procedure to draw shocks from the fitted residuals and is robust to the “tail risks” inherent to probability distributions with fat tails and skewness. One interesting result is that the confidence bands of the debt forecasts based on the bootstrap are tighter than those based on Monte Carlo. Relative to the associated normal density, the histogram of fitted residuals has more probability mass around zero, more probability mass in few extreme values, and less probability mass in medium-sized values. Thus, while it may occasionally draw extreme shocks, the bootstrap most often draws small shocks relative to those of the normal distribution. This makes confidence bands tighter under the bootstrap approach.

The proposed methodology alleviates most of the aforementioned problems. Projections are based on an econometric model that summarizes the co-movements between the key variables and takes into account the uncertainty associated with them. The simulations are computed by drawing shocks from the estimated residuals and, therefore, are based on the “typical” shocks faced by low income countries. Moreover, the issue of identification of structural shocks is irrelevant because we are interested in the usual bundle of shocks hitting low income countries, not in identifying the debt response to some structural shock. In addition, by averaging the behavior of many countries, the methodology is likely to reduce the bias due to poor data quality in low income countries, like too few observations, measurement errors, bad reporting, and so forth.

While aimed at capturing the behavior of a generic low income country, the methodology allows for partial heterogeneity through the introduction of country-specific intercepts (or fixed effects) and exogenous variables. The fixed effects allow the long run value of the determinants of debt to differ across countries. The exogenous variables provide additional heterogeneity. Admittedly, the degree of heterogeneity obtained with these mechanisms could be limited. The methodology, however, is a compromise between the rigidity imposed by the panel vector autoregression and the problems associated with the lack of adequate data in low income countries. For example, the methodology allows performing debt sustainability

analysis even in countries with scarce or no data. The analyst could use external information based, for example, on experts' opinions or from similar countries to propose a reasonable guess of the long run values of the determinants of debt. Given the guess, the analyst can recover the associated country-specific intercept. Moreover, the analyst can also study the impact of reforms that affect the long run values of the determinants of debt. Such reforms are manifested as changes in the country-specific intercept.

In performing these experiments, however, the analyst must be aware that there is a feasibility constraint linking the long run values of the determinants of debt with the long run level of external debt. While the dynamics of debt during the transition could take any form, the choice of the fixed effect determines the level to which foreign debt will converge to in the long run. The link between the long run value of debt and its determinants is not a characteristic of the methodology proposed in this paper but the result of a feasibility constraint that holds independently of the approach used to compute the debt trajectories.

The literature that followed Garcia and Rigobon (2005) is vast.<sup>5</sup> There are, however, some studies that proposed interesting variations to the basic methodology. Celasum, Debrun, and Ostry (2007) incorporate a policy reaction function aimed at capturing the endogenous response of the fiscal surplus to the state of the economy. Frank and Ley (2009) introduce structural breaks in the parameters of a vector autoregression and also use a bootstrap procedure to analyze fiscal sustainability. Arizala et al. (2009) propose combining vector autoregressions with external forecasts to arrive at an average forecast of the main determinants of debt. Few papers have proposed applying Garcia and Rigobon's approach to low income countries. IMF (2004) takes averages over twenty low income countries during 1985-1999 and performs univariate autoregressions for each determinant of debt to generate projected debt trajectories. Hostland (2011) estimates univariate autoregressions for single countries to generate predictions for future debt trajectories. To the best of my knowledge, this is the first paper that proposes to pool the observations of several countries to sum-

---

<sup>5</sup>This methodology has received several names in the literature, including "risk management approach to debt sustainability," "debt sustainability fan charts," and "stochastic simulation methods."

marize the characteristics of a generic low income country but allowing simultaneously for country-fixed effects, country-specific exogenous variables, non-normal residuals, and flexible dynamics using a panel vector autoregression with exogenous variables.

The paper proceeds as follows. Section 2 discusses the basic framework for a single country. Section 3 describes how to implement the methodology using a panel of time series - cross section data. This section also provides a detailed analysis of a fictitious country to illustrate the issues raised above. Section 4 considers Senegal as a case study and briefly compares the proposed methodology with the approach currently used by the IMF and the World Bank. Section 5 provides robustness checks and Section 6 concludes.

## 2 The basic framework

This section describes the basic framework and identifies key variables that affect the dynamics of external debt. To simplify the exposition, this section focuses on a single country. Details of the implementation and how to cope with the data problems of low income countries are discussed in the next section.

The methodology is an extension of the approach proposed by Garcia and Rigobon (2005), originally designed to study the sustainability of public debt, to the study of the sustainability of external debt. The idea is to write a debt accumulation equation, measuring and estimating a stochastic process for the variables that determine the evolution of debt, and performing Monte Carlo simulations to compute a number of statistics on projected debt trajectories. One contribution of this paper is to replace the Monte Carlo simulation step with a bootstrap procedure based on re-sampling from the set of estimated residuals. This modification makes the methodology robust to the “tail risks” inherent to probability distributions with fat tails and skewness.

By definition, the international investment position of a country at time  $t$ ,  $IIP_t$ , equals the position at time  $t - 1$ ,  $IIP_{t-1}$ , plus the current account balance, the amount of debt relief, aid flows, grants, and valuation effects consisting in the change in the value of assets



hold abroad minus the change in the value of assets held by foreigners. Formally,

$$IIP_t = IIP_{t-1} + CA_t + \omega_t,$$

where  $CA_t$  is the current account balance and  $\omega_t$  is the contribution of the remaining items. This equation implies that foreign debt evolves according to (Appendix A provides details)

$$d_t = \frac{1 + r_t}{(1 + g_t)(1 + \pi_t)} d_{t-1} - m_t - f_t + v_t, \quad (2.1)$$

where  $d_t$  is the ratio of external debt to GDP;  $r_t$  is the implicit interest rate on external debt;  $g_t$  is the growth rate of real GDP;  $\pi_t$  is the growth rate of the GDP deflator measured in U.S. dollars;  $m_t$  is the non-interest current account divided by GDP;  $f_t$  denotes net FDI flows as a fraction of GDP; and  $v_t$  includes debt relief, aid flows, grants, and the change in portfolio investment, financial derivatives, and international reserves, all measured as a fraction of GDP.<sup>6</sup> From now on,  $v_t$  will be referred to as a “debt shock.” Changes in any of the variables  $Y_t = \{g_t, \pi_t, r_t, m_t, f_t, v_t\}$  lead to changes in the ratio of foreign debt to GDP.

The debt shock  $v_t$  is difficult to measure, especially in low income countries. Thus, given data on the variables  $\{d_t, g_t, \pi_t, r_t, m_t, f_t\}$ , equation (2.1) can be used to recover the realized value of  $v_t$  as a residual,

$$v_t = d_t - \frac{1 + r_t}{(1 + g_t)(1 + \pi_t)} d_{t-1} + m_t + f_t.$$

The idea of the methodology is to estimate a flexible stochastic process for  $Y_t$ , and then using the estimated process to compute statistics on the evolution of the foreign debt-GDP ratio derived from equation (2.1). For example, the average or median debt trajectory over a number of years, confidence intervals around these point estimates, the probability that

---

<sup>6</sup>The implicit interest rate  $r_t$  is defined as total interest payments on external debt during period  $t$  divided by the stock of external debt in period  $t - 1$ . The non-interest current account  $m_t$  is defined as the current account plus interest payments on external debt.

debt-GDP ratio will cross certain threshold, and so forth. To that end, I assume that  $Y_t$  is a multivariate stochastic process that evolves according to

$$Y_t = \alpha + \sum_{j=1}^p \Theta_j Y_{t-j} + \sum_{h=0}^q \Phi_h X_{t-h} + \varepsilon_t \quad (2.2)$$

where  $t = 1, \dots, T$  is a time index,  $\alpha$  is a  $(6 \times 1)$  vector, the  $\Theta_j$  are fixed  $(6 \times 6)$  matrices on lagged endogenous values,  $X_t$  is a  $(k \times 1)$  vector of exogenous variables, the  $\Phi_h$  are  $(6 \times k)$  matrices on current and lagged exogenous variables, and  $\varepsilon_t$  is a  $(6 \times 1)$  vector of independent and identically distributed (i.i.d.) random shocks with mean zero and covariance matrix  $\Omega$ . Importantly, the shocks  $\varepsilon_t$  could come from any probability distribution.

Specification (2.2) allows for rich dynamics on the determinants of foreign debt, including feedback effects between endogenous variables and interactions of endogenous variables with current and lagged exogenous variables. To compute statistics on foreign debt, the researcher estimates (2.2) and then performs a large number of simulations of length  $\bar{T}$  (the relevant horizon) coupled with the debt accumulation equation (2.1) to generate many debt trajectories. The statistics are then computed by taking sample averages on the simulated data. To implement each simulated path, the researcher draws histories of shocks  $\{\varepsilon_t\}_{t=t_0+1}^{t_0+\bar{T}}$ , where  $t_0$  is the initial period. Next, given a path for the exogenous variables  $X_t$  (more on this below), equation (2.2) is used to compute  $\{Y_t\}_{t=t_0+1}^{t_0+\bar{T}}$ . Finally, given the simulated series, the researcher computes  $\{d_t\}_{t=t_0+1}^{t_0+\bar{T}}$  based on equation (2.1).

The standard implementation of the methodology uses a Monte Carlo method. The researcher takes a stand on the probability distribution of the shocks  $\varepsilon_t$ , usually the normal distribution, and draws shocks from the proposed distribution. A drawback of this approach is that the researcher could choose the wrong distribution. Indeed, in the case of low income countries, the estimation of (2.2) lead to errors that are far from normally distributed: estimated residuals have substantial excess kurtosis (fat tails) and skewness (see Subsection 3.2). To cope with this problem, I replace the Monte Carlo step with a bootstrap procedure.

The researcher estimates (2.2) and then computes the fitted residuals  $\{\widehat{\varepsilon}_t\}_{t=1}^T$ . The required shocks are then drawn with replacement from the set of fitted residuals. This makes the methodology robust to residuals with fat tails and skewness.<sup>7</sup>

A second extension of the specification (2.2), relative to most of the literature, is the inclusion of exogenous variables affecting the dynamics of the endogenous variables. This extension could be important as several variables, arguably exogenous to low income countries (like world GDP growth, commodity prices, world interest rates, or even the terms of trade), are likely to have a non-trivial impact on the endogenous variables. The presence of exogenous variables requires proposing a stochastic process for  $X_t$  independent of that for  $Y_t$ . Thus, in implementing the methodology, I assume that  $X_t$  evolves according to

$$X_t = \beta + \sum_{j=1}^k \Psi_j X_{t-j} + \xi_t \quad (2.3)$$

where  $\beta$  is a  $(k \times 1)$  vector,  $\Psi_j$  are matrices on lagged values, and  $\xi_t$  is a  $(k \times 1)$  i.i.d. shock with zero mean and covariance matrix  $\Sigma$ . To compute trajectories for the exogenous variables, I draw samples assuming that  $\xi_t$  is normally distributed. While it could be possible to use a bootstrap procedure analogous to that describe above, I follow a Monte Carlo procedure for reasons explained below.

### 3 Implementing the methodology using pooled data

The simulation approach has been applied mostly to emerging market economies and advanced countries. In these countries, there is typically sufficiently long time series at a quarterly frequency that makes feasible the estimation of (2.2). Unfortunately, it is virtually impossible to build the required quarterly data set for any low income country. Even

---

<sup>7</sup>The bootstrap procedure helps with tail events as long as they are actually realized in sample. It might happen that these events are never realized in the observed sample, in which case the bootstrapping procedure will not be able to capture them. The sample used in this paper, however, is relatively large with many country-year observations. Moreover, several “extreme” events that do not fit a normal distribution are actually observed in sample.

yearly data are scarce and of dubious quality. Although some low income countries have annual data dating back to 1971, time series with less than 40 observations are insufficient to estimate the process (2.2) in a reliable way. For example, even with 40 observations per equation, a vector autoregression with six endogenous variables, a constant, one lag on the endogenous variables, and no exogenous variables has 63 parameters to estimate with 240 observations, less than 4 observations per parameter.

To cope with this problem, I pool data from several low income countries allowing for partial heterogeneity in the form of country fixed-effects and country-specific exogenous variables, but assuming common slope coefficients and covariance matrix across countries. Thus, I replace the specification (2.2) with a vector autoregression applied to a panel of cross-country and time series data (Panel VARX) represented by

$$Y_{i,t} = \alpha_i + \sum_{j=1}^p \Theta_j Y_{i,t-j} + \sum_{h=0}^q \Phi_h X_{i,t-h} + \varepsilon_{i,t}. \quad (3.1)$$

Here, countries are indexed by  $i = 1, 2, \dots, N$ ; the time index is  $t = 1, 2, \dots, T_i$ , where  $T_i$  denotes the number of usable observations per country; and  $p$  and  $q$  denote the number of lags on the endogenous and exogenous variables respectively. Thus, the total number of observations is  $T = \sum_{i=1}^N T_i$ . In addition,  $\alpha_i$  is a vector of country fixed-effects and the residuals  $\varepsilon_{i,t}$  are i.i.d. shocks satisfying  $E(\varepsilon_{i,t}) = \mathbf{0}$ ,  $E(\varepsilon_{i,t}\varepsilon'_{i,t}) = \Omega$  for all  $i$  and  $t$ ,  $E(\varepsilon_{i,t}\varepsilon'_{i,s}) = \mathbf{0}$  for all  $i$  and  $t \neq s$ , and  $E(\varepsilon_{i,t}\varepsilon'_{j,s}) = \mathbf{0}$  for any  $s, t$  and  $i \neq j$ . The slope coefficients  $\Theta_j$  and  $\Phi_k$ , and the covariance matrix  $\Omega$  are common across countries. Thus, the methodology computes the dynamic response on the endogenous variables of a generic low income country. Note, however, that there are two sources of heterogeneity across countries. The first takes the form of a fixed-effect affecting the intercept of the regression. The second is that the realization and stochastic process followed by the exogenous variables  $X_{i,t}$  could differ across countries.

The endogenous variables are represented by the  $(6 \times 1)$  vector

$$Y_{i,t} = \begin{bmatrix} \text{Real GDP growth}_{i,t} \\ \text{Growth of GDP deflator in US dollars}_{i,t} \\ \text{Implicit interest rate}_{i,t} \\ \text{Non-interest current account/GDP}_{i,t} \\ \text{Net flow of FDI/GDP}_{i,t} \\ \text{Debt shock}_{i,t} \end{bmatrix}.$$

The set of exogenous variables is divided in two groups. The first is a group of common variables to all countries and includes (i) World GDP growth, (ii) a world interest rate proxied by the U.S. one year constant maturity treasury rate, and (iii) the logarithm of the price of oil. The second is a set of country-specific variables. Choosing a country specific exogenous variable is problematic due to endogeneity concerns. The terms of trade is one variable that is arguably exogenous to developing countries. This is a usual assumption made in the literature and is based on the observation that developing countries, being small relative to the rest of the world, have a negligible impact on the relative prices they face in world markets (for example, Broda, 2004; Fomby, Ikeda, and Loayza, Forthcoming). Following this literature, I include the logarithm of the terms of trade as an exogenous variable.<sup>8</sup> In sum, the vector of exogenous variables used in this paper is given by

$$X_{i,t} = \begin{bmatrix} \text{Log of terms of trade}_{i,t} \\ \text{World GDP growth}_t \\ \text{U.S. one year constant maturity treasury rate}_t \\ \text{Log price of oil}_t \end{bmatrix}.$$

Since the vector  $X_{i,t}$  includes a country-specific variable, the parameters of the process

---

<sup>8</sup>I use the log-levels of the price of oil and the terms of trade because, as shown in Section 3.3 below, it is possible to reject the null hypothesis of a unit root in both of these variables.

(2.3) must also be indexed by country  $i$ , or

$$X_{i,t} = \beta_i + \sum_{j=1}^k \Psi_{i,j} X_{i,t-j} + \xi_{i,t}. \quad (3.2)$$

It is well known that fixed-effect least squares estimators of dynamic panel models (also called least squares dummy variables estimator, or LSDV) lead to inconsistent estimates when the time dimension  $T_i$  is short and fixed, even if the cross-section dimension  $N$  increases to infinity (Nickell, 1981). As  $T_i$  grows large, however, the bias decreases and disappears as  $T_i$  goes to infinity. In practical terms, however,  $T_i$  of the order of 20-30 is usually enough to make the bias small. The basic data set used in this paper includes 76 low income countries with data going back to 1971. The panel, however, is unbalanced. Thus, whether the sample is long enough is an empirical question. To cope with this issue, I implement a version of the bootstrap bias correction algorithm originally proposed by Pesaran and Zhao (1999) and recently extended by Tanizaki, Hamori, and Matsubayashi (2006), Everaert and Pozzi (2007), and Fomby, Ikeda, and Loayza (Forthcoming). The interested reader is referred to these papers for more details on the bias correction procedure.<sup>9</sup>

### 3.1 Data

The data consist of an unbalanced panel of 76 low income countries over the period 1971-2007 for which there is enough information to construct uninterrupted time series for the endogenous variables  $Y_{it}$ . Table 1 provides a list of the countries and the years for which there are data for the entire vector of endogenous variables. Data are obtained from the World Bank's World Development Indicators and the IMF's World Economic Outlook databases.

---

<sup>9</sup>Fomby, Ikeda, and Loayza (Forthcoming) implement and extend a version of the bias correction method proposed by Pesaran and Zhao (1999) to Panel VARs. In this paper, instead, I use an iterative procedure similar to those proposed by Tanizaki, Hamori, and Matsubayashi (2006) and Everaert and Pozzi (2007). The basic difference between the two approaches is in the number of iterations in the bias correction algorithm: while Pesaran and Zhao propose a single iteration on the procedure, Tanizaki, Hamori, and Matsubayashi and Everaert and Pozzi propose iterating on an equation mapping regression coefficients into updated regression coefficients. The bias corrected estimator is the fixed point of that equation.

The variables and sources are presented in Table 2, and all growth rates are reported as log-differences.

Table 3 reports summary statistics of the raw data, including the debt shock constructed using equation (2.1). The mean and median growth rates of GDP are 3.6 and 4.0 percent respectively, but there is substantial heterogeneity across countries, as reflected in a standard deviation of 5.4 percentage points. Moreover, the minimum and maximum values observed for the growth rate of real GDP are -70 and 30 percent, both corresponding to Rwanda during 1994 and 1995 respectively. The average and median growth rates of the debt shock are zero, but with a large volatility. The minimum debt shock corresponds to a large debt relief episode occurred in Nicaragua, in 1996. Finally, the concessionality of the debt in low income countries can be inferred by looking at the implicit interest rate. On average, low income countries pay an interest of about 2.6 percentage points per year on their external debt, with a relatively low standard deviation, of just 2 percentage points.

Table 4 reports the contemporaneous correlation of all endogenous and endogenous variables. Focusing on the first column, one observes that GDP growth tends to be negatively correlated with real exchange rate depreciations (as reflected in its negative correlation with the growth of the GDP deflator in U.S. dollars), with net FDI inflows, with world growth, and with the price of oil. In addition, GDP growth is negatively correlated with both interest rate measures, particularly so with the U.S. treasury rate. Moreover, there is a large correlation between the implicit interest rate and the U.S. treasury rate, of about 46 percent. This suggests that, although debt in low income countries contains a substantial concessional component, it also responds to market forces. The table also shows a negative correlation between the U.S. interest rate and FDI flows: periods with high interest rates are periods with relatively low FDI flows, also consistent with the view that market forces do play significant role in low income countries. Overall, two lessons can be learned from Table 4: first, it is important to take into account the co-movements between the variables that drive the evolution of external debt, and second, it is important to incorporate exogenous variables

into the analysis.

### 3.2 Estimation of the Panel VARX

The lag structure of the panel VARX was chosen according to the Schwarz's Bayesian information criterion (SBIC) and the Akaike information criterion (AIC). These are two standard goodness of fit criteria that select the lag length of dynamic models by adding a penalty term to the likelihood value that increases with the number of parameters. The preferred model is the one with lowest value of the information criterion. Table 5 reports the SBIC and AIC values for different estimating models. In the table,  $p$  and  $q$  represent the number of lags in the endogenous and exogenous variables respectively. The SBIC criterion selects the most parsimonious model with one lag in the endogenous variables and no lags in the exogenous variables. The AIC criterion selects a model with two lags in both the endogenous and exogenous variables. To keep the model as parsimonious as possible, I use the lag structure selected by the SBIC criterion. (As a robustness check, Section 5 considers the model selected by the AIC criterion.) The database is reduced to 72 countries with the required information once we include the exogenous variables and proposed lag structure.

One contribution of this paper is to build a methodology consistent with fat tails and skewness in the residuals of the equation (3.1). Figure 1 displays histograms of the estimated residuals together with normal density functions with identical mean and variances. If residuals are well approximated by a normal distribution, the histograms and the normal densities should be close to each other. They are not. The histograms have fatter tails than the normal density and, in some cases, one can observe some skewness as well. To complement the graphical analysis, I performed six univariate and joint tests of (i) normality, (ii) no excess kurtosis, and (iii) no skewness of the residuals based on the tests proposed by Urzua (1997) (Table 6). In all cases, normality, no excess kurtosis, and no skewness are rejected with extremely high confidence in both the univariate and joint tests. Under the null hypothesis of joint normality of residuals, the asymptotic distribution of the test statistic is



chi-square with 12 degrees of freedom. The estimated statistic is about 390000, leading to rejection of the null hypothesis with enormous confidence. Multivariate tests of no excess kurtosis and no skewness are also rejected with great confidence. Moreover, the kurtosis statistic is two orders of magnitudes larger than the skewness statistic. This suggests that the huge value of the joint normality test statistic is mostly due to fat tails in the distribution of residuals. Regarding univariate tests, the data also reject normality, no excess kurtosis, and no skewness for every residual. These results reinforce the need to use the bootstrap procedure discussed above to perform the simulations.

Table 7 reports estimation results for the baseline specification. The upper panel reports estimates based on the LSDV estimator; the lower panel, estimates based on the bias corrected procedure. The first two columns show the estimated coefficients  $\Theta_1$  and  $\Phi_0$ . The matrices on the third column report the estimated standard deviations of the residuals (on the main diagonal) and the correlation coefficients between estimated residuals (on the off-diagonals). The two estimators deliver coefficients of similar magnitude except for those in the main diagonal of  $\Theta_1$ . The bias corrected estimator implies more persistence in the  $Y_t$  process than the LSDV estimator.

### 3.3 Country-specific information and implementation details

Additional pieces of information are still needed to apply the methodology in a particular country: the country-specific intercept and the projections of the exogenous variables.

The Panel VARX provides estimates of country-specific intercepts  $\alpha_i$ . These estimates are related to the mean values of the endogenous variables. In particular, taking unconditional expectation in equations (2.3) and (3.1), and rearranging gives

$$\bar{Y}_i = \left( \mathbf{I}_6 - \sum_{j=1}^p \Theta_j \right)^{-1} \left[ \alpha_i + \left( \sum_{h=0}^q \Phi_h \right) \left( \mathbf{I}_4 - \sum_{j=1}^k \Psi_j \right)^{-1} \beta \right] \text{ for all } i, \quad (3.3)$$

where  $\bar{Y}_i$  is the unconditional expectation of the endogenous variables in country  $i$  and  $\mathbf{I}_s$

denotes an identity matrix of dimension  $s$ . This equation relates the parameters of (2.3) and (3.1) to the long run averages of the endogenous variables  $Y_{i,t}$ . Of course, because samples are finite, the estimated country-specific intercept will be linked to historical averages instead of population averages.

A baseline analysis proceeds as follows. If the proposed country is in the database, the simulations are run using the estimated fixed effect for that country. If the country is not in the database or if the analyst distrusts the estimated historical averages, she could use outside information (like experts' opinions or data from a similar country) to estimate or guess a long run value for the endogenous variables, say  $\tilde{Y}$ . Then, she could use (3.3) to find the intercept as

$$\tilde{\alpha} = \left( \mathbf{I}_6 - \sum_{j=1}^p \Theta_j \right) \tilde{Y} - \left( \sum_{h=0}^q \Phi_h \right) \left( \mathbf{I}_4 - \sum_{j=1}^k \Psi_j \right)^{-1} \beta. \quad (3.4)$$

Finally, simulations are based on the estimated parameters and the implied intercept  $\tilde{\alpha}$ .

The analyst could also perform debt sustainability analyses under reform scenarios. For example, suppose the analyst believes that some reform will increase the long run flow of FDI to the country (and, perhaps indirectly, other endogenous variables as well). Then, she could replace the vector of historical averages with a new vector  $\tilde{Y}$  reflecting the long run expectations of the reform. Equation (3.4) is then used to recover the intercept  $\tilde{\alpha}$  to be used in the debt sustainability analysis.

In such cases, however, the analyst must be aware that there is a feasibility constraint relating the long run values of the endogenous variables with that of foreign debt. In effect, taking unconditional expectations to both sides of equation (2.1) and rearranging leads to

$$\bar{d} = \frac{(1 + \bar{g})(1 + \bar{\pi})}{(1 + \bar{r}) - (1 + \bar{g})(1 + \bar{\pi})} (\bar{m} + \bar{f} - \bar{v})$$

where a 'bar' above a variable denotes its unconditional expectation. Thus, while the dynamics of external debt could vary during the transition to the steady state, the fixed effect

determines the level to which foreign debt will converge to in the long run. This link between the long run value of the endogenous variables and the long run value of debt is the result of a feasibility constraint that holds independently of the methodology used to perform debt sustainability analysis.

One could argue that it is strange to implicitly fix the long run level of debt through the choice of the country fixed effect in an exercise whose objective is precisely to analyze the sustainability of debt. The estimates obtained in the next section, however, imply that external debt tends to converge to its long run value in 70 years or more. During the relevant horizon (20 years or, preferably, less), the forecast levels of foreign debt are usually quite different from those long run values.

The analyst also needs to estimate projections for the exogenous variables based on the process (3.2). This specification could be difficult to estimate for each country of interest due to data limitations. The following assumptions are imposed. First, I assume that the (log) of the terms of trade follow a univariate autoregressive process independent of the other exogenous variables. To perform the analysis for countries with minimal or no data on the terms of trade, the analyst could use estimates from similar countries or simply assume a process for it. Second, I assume that world GDP growth and the U.S. interest rate follow a bivariate vector autoregression and that (log) oil prices follow a separate autoregressive process. This is done for convenience and not necessarily for realism. In effect, oil prices behave quite differently before and after the mid 1970s. Thus, I use relatively long time series (starting in 1962) to estimate a VAR for world GDP growth and the U.S. interest rate, and a shorter time series to estimate the process for the price of oil and the terms of trade.

A standard augmented Dickey Fuller test rejects the null hypothesis of a unit root in the logarithm of the price of oil over 1974-2010. According to the SIC and AIC information criteria, an autoregressive process with one lag is enough to describe the dynamics of the price of oil. The points estimates of the constant and coefficient on lagged oil price are 0.33 and 0.91 respectively. The estimated standard deviation of the residuals is 0.27 and the

Durbin Watson statistic is 2.08, consistent with the absence of serial correlation in the fitted residuals. On the other hand, the two information criteria select a vector autoregression of order 2 for the growth rate of world GDP and the U.S. interest rate.

Finally, to obtain projections for the exogenous variables, I draw samples assuming that the residuals of the process (3.2) are normally distributed instead of following the bootstrap approach. The reason for this choice is that many countries have short time series for their country-specific exogenous variables (20 observations or less). It could be very misleading to draw from such a small set of fitted residuals.

### 3.4 A detailed example

This subsection implements the methodology using a fictitious country. First, I perform a basic analysis of future debt trajectories and asses risks of debt distress, defined as the probability that future debt trajectories cross certain thresholds. Second, I discuss how to implement the analysis under a reform scenario that permanently increases the flow of FDI in the long run. Finally, I use this example to illustrate that confidence bands generated with the bootstrap are tighter than those obtained under Monte Carlo.

To perform the experiment, I need to select a process for the logarithm of the terms of trade, the initial values for  $X_t$  and  $Y_t$ , and the country-specific intercept. I assume that the logarithm of the terms of trade follows the process

$$\log ToT_t = 0.68 + 0.85 \log ToT_{t-1} + u_t; \quad u_t \sim N(0, 0.15^2).$$

This process is roughly consistent with the observed evolution of terms of trade in low income countries. The long run mean and standard deviation of the terms of trade implied by the above process are 100 and 29.1 respectively.

The second and third columns of Table 8 display the initial conditions used in the example. At time zero, the country has a level of foreign debt of 45 percentage points of GDP,

a growth rate of real GDP of 3 percent, a growth rate of the GDP deflator measured in U.S. dollars of 5 percent, and an implicit interest rate of 2 percent. The non-interest current account and FDI are -7 and 3 percentage points of GDP respectively. These numbers imply that the debt shocks is about -1.4 percentage points of GDP. The terms of trade is initialized at log 100, and the price of oil is assumed to be 100 dollars per barrel. In addition, I assume that at times  $t = -1, 0$  world growth is 2 percent and the U.S. interest rate is 4 percent. To set  $\alpha$ , I use equation (3.4) and assume that the long run values of the six endogenous variables are as displayed in the last column of Table 8. The remaining numbers in that column are the implied long run values of the exogenous variables and of the debt-to-GDP ratio as derived from equations (2.1) and (2.3).<sup>10</sup>

The upper left panel of Figure 2 displays the projected histories of the debt-to-GDP ratio over a 10 year horizon. These projections are based on 100000 simulations of length 10, starting from the initial conditions in Table 8 and drawing shocks according to the bootstrap procedure. The bold line is the median debt-to-GDP trajectory and the dashed lines are the lower and upper quartiles of the implied distribution. The shaded areas denote percentiles of the debt-to-GDP ratio at 5 percent increment. There are several things to note. Ignoring uncertainty can be misleading in the debt sustainability analysis. In particular, one can interpret the median evolution as the baseline projection of the debt-to-GDP ratio over the proposed period. In this example, the median debt trajectory declines to 35 percentage points of GDP over the proposed horizon, suggesting a low risk of debt distress. Results are different, however, once we take into account the uncertainty involved in the projections. Percentile bands are wide. One can find many trajectories with sufficiently bad shocks that drive the debt-to-GDP ratio to over 100 percent. Admittedly, the 95th percentile might be an overly conservative bound to consider. Still, the upper quartile increases from the initial 45 percent to 62 percentage points over the simulation horizon, a non-trivial increase.

---

<sup>10</sup>The long run average of the estimated process for the price of oil is 33.5. The joint process for world growth and the U.S. interest rate delivers long run values of 3.5 and 6 percent respectively. Evaluating (2.1) at the steady state gives a long run value of debt-to-GDP of 0.69.

A measure of “risk of debt distress” can be constructed by computing the probabilities, conditional on information at time zero, that the debt-to-GDP ratio will cross some threshold over the next years. The upper right panel of Figure 2 displays these probabilities for debt thresholds of 60, 80, and 100 percentage points of GDP. I compute these probabilities at each time horizon  $t = 1, 2, \dots, 10$  by counting the number of debt histories that cross the proposed bound at each horizon and dividing it by 100000. In this exercise, the probability that foreign debt increases to 60 percent of GDP in ten years is 26 percent, to 80 percent of GDP is 15 percent, and to 100 percent of GDP is 9 percent.

Figure 3 reports the evolution of the six determinants of debt. These projections give an idea of the variables that explain the large confidence bands of the debt trajectories. The GDP deflator in U.S. dollars is the most volatile variable, followed by the debt shock, the non-interest current account, and GDP growth. The implicit interest rate and FDI are less volatile. Moreover, these confidence bands are consistent with the standard deviations reported in Table 7. Note, however, that shocks tend to come in bundles. For example, positive shocks to the non-interest current account tend to be associated with negative shocks to FDI. This, of course, reflects that estimated residuals are reduced form shocks of some underlying structural shocks that remain unidentified by the proposed methodology.

The lower panel of Figure 2 shows results for a counterfactual reform that increases the long run flow of FDI by one percentage point of GDP. The proposed reform has a large impact on projected trajectories of debt. For example, the median forecast of foreign debt decreases to 27 percentage points of GDP in 10 years, 8 percentage points smaller than before the reform. The upper quartile increases only to 54 percentage points of GDP, 8 percentage points lower than before the reform. Likewise, the probabilities that debt will cross any of the proposed thresholds decline substantially after the reform.

Finally, Figure 4 compares statistics computed with the bootstrap procedure with those based on Monte Carlo assuming normal shocks with a covariance matrix equal to that of the fitted residuals. Median debt trajectories are fairly similar under both methods. Sim-

ulations based on Monte Carlo, however, predict a somewhat lower debt-to-GDP ratio at the end of the projection horizon. This might happen because the bootstrap distribution is right-skewed. The remaining plots in Figure 4 show inter-percentile ranges of projected debt-to-GDP ratios. For example, the lower left panel reports the inter-quartile range of debt forecasts. The inter-quartile range is wider under the Monte Carlo approach. The same is true for the 60th-40th inter-percentile range. The reason for this result is the following. Relative to the associated normal density, the histogram of fitted residuals has more probability mass around zero, more probability mass in few extreme values, and less probability mass in medium-sized values. Thus, while it may occasionally draw extreme shocks, the bootstrap most often draws small shocks relative to those of the normal distribution. This makes confidence bands tighter under the bootstrap approach. On the other hand, the 95th-5th inter-percentile ranges are similar in both methods. This might be reflecting that these percentiles are capturing the extreme realizations occasionally drawn by the bootstrap.

## 4 A case study: Senegal

This section considers the case of Senegal to highlight some issues in implementing the methodology. It is argued that following a mechanical approach could be misleading. To obtain reasonable results, the analyst needs to consider carefully the choice of the country-specific intercept. Failure to do so could lead to overly optimistic or overly pessimistic debt scenarios. This section also compares the predictions obtained under the proposed methodology with those of the baseline IMF–World Bank’s debt sustainability analysis (DSA).

In 1996, the IMF and the World Bank launched the *Heavily Indebted Poor Countries* (HIPC) initiative which consisted in providing debt relief to countries with unmanageable debt burdens. In 2004, Senegal was granted 850 million U.S. dollars in debt service relief, a large fraction of which was implemented immediately. Senegal’s external debt declined substantially, reinforcing a trend that started in 2000 (top panel of Figure 5). Foreign debt

declined from 82 to 34 percentage points of GDP between 2000 and 2006, with the bulk of the drop between 2004 and 2006. Since 2006, however, the debt-to-GDP ratio began to increase, reaching slightly over 50 percentage points of GDP in 2010.<sup>11</sup>

Panel B of Figure 5 displays the evolution of the six determinants of foreign debt, as identified by equation (2.1). The average growth rate of real GDP between 2001 and 2010 was 4 percentage points, although with significant volatility. The GDP deflator in U.S. dollars (that is, the reciprocal of the real exchange rate) was highly volatile, and the country had a persistent current account deficit net of interest payments. In addition, the large negative values of the debt shock during 2005 and 2006 (-9 and -26 percent respectively) reflect the realization of the debt relief agreed under the HIPC initiative. FDI flows remained roughly constant, at around 1.5 percentage points of GDP, and the implicit interest rate was virtually constant at less than one percentage point.

The top panel of Figure 6 displays the evolution of foreign debt / GDP and the probabilities that debt will cross the proposed thresholds during 2011-2020. Here, the country-specific intercept is set at its estimated value based on historical data. The median foreign debt trajectory decreases from 52 to just over 5 percentage points of GDP by 2020. Likewise, the upper quartile of foreign debt decreases to 27 percentage points of GDP over the same horizon. These trajectories imply that the probability of foreign debt increasing to 60 percentage points of GDP during the next ten years is always smaller than 8 percentage points. Similarly, the probabilities that debt will increase to more than 80 or 100 percent of GDP never exceed 4 and 2 percentage points respectively. These predictions for foreign debt are, to a large extent, driven by the proposed intercept. Evaluating equation (3.3) at the estimated intercept implies very optimistic long run values of the endogenous variables: real GDP growth is 4 percent, the growth rate of the GDP deflator in U.S. dollars is 5 percent, the flow of FDI is 3 percentage points of GDP, and the debt shock is -4.5 percentage points of

---

<sup>11</sup>Throughout this section, the data for the debt-to-GDP ratio and the endogenous variables are taken from the IMF–World Bank May 2011 DSA (IDA and IMF, 2011).



GDP. These averages induce a strong decline in trajectories of foreign debt.<sup>12</sup>

These estimated long run values, however, do not seem reasonable. For example, the implied long run value for the debt shock is highly affected by the debt relief episode associated with the HIPC initiative. In addition, it is difficult to believe that the yearly rate of appreciation of the real exchange rate in Senegal will be, on average, 5 percentage points for the indefinite future. Therefore, it is important to be particularly careful with the long run values of the determinants of debt implied by the proposed country-specific intercept. To make this point more clear, the lower panel of Figure 6 displays results analogous to those in the top panel, but setting the long run values of the endogenous variables in Senegal equal to the pooled averages across time and countries.<sup>13</sup> Results are quite different from those in the top panel. For example, the median debt trajectory now decreases to only 29 percentage points of GDP. Moreover, the probability that debt will increase to 60 percentage points of GDP by 2020 is now over 20 percent, more than 12 percentage points larger than the probability displayed in the top panel.

In sum, this exercise highlights the importance of having reasonable and accurate forecasts for the long run values of the main determinants of debt. Historical data is probably not the best way to do that, given the changing environment in low income countries. Here, the insight of experts knowledgeable of the country's idiosyncrasies could be extremely valuable. Alternatively, one could perform debt sustainability analysis under different long run scenarios to assess the robustness of the results.

## 4.1 Comparison with the IMF–World Bank DSA

This subsection explores the consequences of ignoring the co-movements between the determinants of debt by comparing the predictions of the stress tests of a typical IMF–World Bank DSA (IDA and IMF, 2011) with those obtained using the methodology proposed in

---

<sup>12</sup>The non-interest current account is -6 percentage points of GDP, inducing an increase in foreign debt. This force, however, is not enough to neutralize the strong debt-reducing force of the other variables.

<sup>13</sup>The implied country-specific intercept follows from equation (3.4).

this paper. Because the IMF–World Bank framework ignores uncertainty, some changes are needed to make the comparison possible.

The Senegal DSA includes baseline projections for external debt, real GDP growth, implicit interest rate, growth rate of GDP deflator in U.S. dollars, non-interest current account, and FDI flows until 2030. Given these values, I recover the debt shock as explained in Section 2. Next, I obtain the residuals  $\tilde{\varepsilon}_t$  for  $t = 2011, 2012, \dots, 2030$  that make the predictions from the IMF–World Bank DSA consistent with the econometric model characterized by equations (3.1) and (3.2). While still ignoring uncertainty, this approach is equivalent to view the projections from the DSA as being derived from a particular realization of the econometric model (3.1)–(3.2). Finally, I compute the standard deviation of the six determinants of debt using their values over 2001–2010.

To compute the stress tests, I proceed as follows. The stress test according to the IMF–World Bank framework consists of adding a negative shock equal to one standard deviation of the corresponding residual during 2011–2012. For example, in the case of GDP growth, I subtract one standard deviation from the baseline projection only during the years 2011 and 2012. The remaining years and variables are set as in the baseline projection. A stress test consistent with the methodology proposed in this paper is more involved. Consider the case of GDP growth. On average, shocks to GDP growth are correlated with shocks to the other variables. To take into account these correlations, I adjust the five remaining residuals in such a way that they are, on average, consistent with the covariance matrix of residuals. Specifically, let  $\varpi$  denote the proposed one standard deviation shock to GDP growth that lasts for two periods and let  $\zeta_t$  for  $t = 2011, 2012, \dots, 2030$  denote a new vector of residuals for the proposed stress test. For  $t = 2013, 2014, \dots, 2030$ , this vector satisfies  $\zeta_t = \tilde{\varepsilon}_t$ . For  $t = 2012$  and 2013, the vector of residuals is given by  $\zeta_{i,t} = \tilde{\varepsilon}_{i,t} + (\Omega_{1,i}/\Omega_{1,1})\varpi$  where sub-index  $i$  denotes the  $i$ th element of the corresponding vector and  $\Omega_{i,j}$  is the  $(i, j)$  element of the estimated covariance matrix of reduced form shocks.<sup>14</sup> Finally, to compute the projected

---

<sup>14</sup>Formally, I project the change in each reduced form shock into the change in the first reduced form shock. This is the same approach proposed by Pesaran and Shin (1998).

values for the determinants of debt, I use the dynamic equations (3.1)–(3.2) evaluated at the residuals  $\zeta_t$ .

Figure 7 reports the baseline projection of foreign debt (solid line), stress tests ignoring co-movements (dashed line), and stress tests taking into account the correlation between the reduced form shocks and feedback effects between the determinants of debt (circled line). Each panel represents a stress test equal to one standard deviation shock to the proposed determinant of debt lasting for two years. The difference between the dashed and circled lines represents the bias induced by ignoring co-movements. In most cases, this difference is non-trivial and, typically, ignoring co-movements underestimates the increase in foreign debt. For example, while a negative shock to FDI barely changes external debt according to the IMF–World Bank stress test, it does induce a sizable increase in foreign debt once the co-movements are taken into account. In sum, this exercise suggests that ignoring co-movements, as currently done in the IMF–World Bank debt sustainability framework, could lead to important biases in the projected trajectories of external debt.

## 5 Robustness checks

This section discusses three alternative specifications to assess the robustness of the baseline estimation. First, I consider the stability of the results to the sample of countries included in the estimation. Second, I discuss results for the model chosen by the AIC criterion, which includes two lags in the endogenous variables and two lags in the exogenous variables. Finally, I re-estimate the model without exogenous variables. The robustness of the results is assessed by comparing predicted debt trajectories and probabilities that debt crosses the proposed bounds in the example economy under the baseline and alternative specifications.

Consider first the stability of the results to the sample of countries used in the estimation. The top panel of Figure 8 displays results for a sub-sample of countries with 15 years of data or more (the sample is reduced from 72 to 49 countries). The remaining panels of the figure

report results for sub-samples with twenty countries deleted at random independently of their number of observations. In all cases, results are similar to those displayed in Panel A of Figure 2, both in terms of projected debt trajectories and probabilities that foreign debt / GDP cross the proposed bounds. Thus, results do not seem sensitive to the particular sample of countries used.<sup>15</sup>

Consider next the predictions of the model selected by the AIC information criterion. This is a model with two lags in the endogenous variables and two lags plus the contemporary effect in the exogenous variables. Results are displayed in the top panel of Figure 9. Predicted debt trajectories and probabilities of debt distress are very similar to those of the baseline specification. Thus, it seems that there is not any significant loss of information in choosing the more parsimonious model selected by the SBIC information criterion.

Finally, results for the econometric specification with no exogenous variables are shown in the lower panel of Figure 9. Here we observe some differences relative to the baseline model. In particular, the median, quartiles, and percentile bands of all debt trajectories are higher than in the baseline specification. This implies that the probabilities of debt crossing the proposed thresholds are also greater than those in the baseline specification. I interpret these findings as reflecting the importance of including the proposed exogenous variables when performing debt sustainability analyses. In this example, failing to do so implies overly pessimistic statements about the projected evolution of foreign debt.

## 6 Concluding remarks

This paper presents an alternative to the current IMF–World Bank debt sustainability framework for low income countries. The proposed methodology alleviates several problems of the current framework. Debt projections are based on a well defined econometric model that takes into account the co-movements between the main determinants of debt and the un-

---

<sup>15</sup>Results are still similar even when deleting thirty countries at random. There are, however, somewhat larger differences relative to the baseline model. This is to be expected as deleting thirty countries implies dropping almost half of the original sample of countries.

certainty associated with them. Simulations are computed by drawing shocks from a set of estimated residuals and, therefore, are based on the typical shocks faced by low income countries—typically associated with a distribution with fat tails and skewness. In addition, by averaging the behavior of many low income countries, the methodology is likely to reduce the bias due to poor data quality in low income countries. As an example, the methodology was applied to a fictitious country and to Senegal. Results in the paper suggest that ignoring the co-movements between the main determinants of debt and the uncertainty associated projected debt trajectories lead to significant biases in these trajectories and, consequently, in the assessment of whether external debt is sustainable or not.

The methodology has some limitations. First, it constraints the degree of cross-country heterogeneity to a country-specific intercept and to country-specific exogenous variables. The latter, however, could follow a different stochastic process for each country. In any case, the methodology is a compromise between the rigidity imposed by the panel vector autoregression and the problems associated with the lack of adequate data in low income countries. Second, the set of reforms that can be analyzed by the methodology is limited. The methodology considers reforms affecting only the long run values of the determinants of debt. Reforms affecting short or medium term dynamics are not captured. This, however, is a problem of any econometric approach analyzing policy evaluation, as Lucas (1976) pointed out. Analyzing the impact of a reform on the entire data generating process requires writing a fully specified structural model, something beyond the scope of this paper.

Finally, if other sources of country-specific forecasts are available, it could be possible to combine these forecasts with those proposed by the current methodology. This could be done by using Bayesian or other model averaging techniques to minimize prediction errors. The combined model would lead to more precise forecasts and, therefore, to tighter confidence bands around median debt trajectories.

## References

- Arizala, Francisco, Christian Castro, Eduardo Cavallo, and Andrew Powell. 2009. "Debt sustainability fan charts: combining multivariate regression analysis and external forecasts." IADB Working Paper.
- Broda, Christian. 2004. "Terms of trade and exchange rate regimes in developing countries." *Journal of International Economics* 63 (1):31–58.
- Celasum, Oya, Xavier Debrun, and Jonathan Ostry. 2007. "Primary surplus behavior and risks to fiscal sustainability in emerging market countries: a "Fan-Chart" approach." *IMF Staff Papers* 53 (3):401–425.
- Everaert, Gerdie and Lorenzo Pozzi. 2007. "Bootstrap-based bias correction for dynamic panels." *Journal of Economic Dynamics and Control* 31 (4):1160–1184.
- Fomby, Thomas, Yuki Ikeda, and Norman V. Loayza. Forthcoming. "The growth aftermath of natural disasters." *Journal of Applied Econometrics*.
- Frank, Nathaniel and Eduardo Ley. 2009. "On the probabilistic approach to fiscal sustainability: structural breaks and non-normality." *IMF Staff Papers* 56 (4):742–757.
- Garcia, Marcio and Roberto Rigobon. 2005. "A risk management approach to emerging market's sovereign debt sustainability with an application to Brazilian data." In *Inflation Targeting, Debt and the Brazilian experience 1999 to 2003*. MIT Press.
- Hostland, Doug. 2011. "What role can stochastic simulation methods play in the DSF?" IMF Working Paper.
- IDA and IMF. 2011. "Senegal: Joint IMF/IDA debt sustainability analysis." <http://www.imf.org/external/pubs/ft/dsa/pdf/dsacr11139.pdf>. [Accessed 21-November-2011].
- IMF. 2004. "Debt sustainability in low income countries: Proposal for an operational framework and policy implications."
- Lucas, Robert Jr. 1976. "Econometric policy evaluation: A critique." *Carnegie-Rochester Conference Series on Public Policy* 1 (1):19–46.
- Nickell, Stephen J. 1981. "Biases in dynamic models with fixed effects." *Econometrica* 49 (6):1417–26.
- Painchaud, Françoise and Tihomir Stučka. 2011. "Stress testing in the Debt Sustainability Framework (DSF) for low-income countries." World Bank Working Paper.
- Pesaran, M. Hashem and Yongcheol Shin. 1998. "Generalized impulse response analysis in linear multivariate models." *Economics Letters* 58 (1):17 – 29.
- Pesaran, M. Hashem and Zhongyun Zhao. 1999. "Bias reduction in estimating long-run relationships from dynamic heterogeneous panels." In *Analysis of Panels and Limited Dependent Variable Models*. Cambridge University Press, 297–322.

- Tanizaki, Hisashi, Shigeyuki Hamori, and Yoichi Matsubayashi. 2006. "On least-squares bias in the AR(p) models: Bias correction using the bootstrap methods." *Statistical Papers* 47 (1):109–124.
- Urzua, Carlos M. 1997. "Omnibus tests for multivariate normality based on a class of maximum entropy distributions." In *Applying Maximum Entropy to Econometric Problems (Advances in Econometrics, Volume 12)*, edited by R. Carter Hill. Emerald Group Publishing Limited, 341–358.
- World Bank and IMF. 2009. "A review of some aspects of the low-income Debt Sustainability Framework."

## A Appendix

This Appendix provides a detailed derivation of equation (2.1). Consider the evolution of the international investment position

$$IIP_t = IIP_{t-1} + CA_t + \omega_t. \quad (\text{A.1})$$

First, decompose the international investment position as  $IIP_t = S_t - D_t$ , where  $D_t$  is the stock of foreign debt and  $S_t$  is the stock of equity-like positions (direct investment, portfolio investment, financial derivatives) and international reserves. Second, define the *non-interest current account* as  $NICA_t = CA_t + INT_t$ , where  $INT_t$  is the country's interest payments on its external debt. Introducing these definitions into the previous equation gives

$$S_t - D_t = S_{t-1} - D_{t-1} + NICA_t - INT_t + \omega_t,$$

or, rearranging,

$$D_t = D_{t-1} (1 + r_t) - NICA_t + S_t - S_{t-1} - \omega_t,$$

where  $r_t = INT_t/D_{t-1}$  is the implicit interest rate paid on external debt.

Furthermore, let  $S_t = P_t - DI_t$ , where  $DI_t$  is the (net) stock of direct investment owned by foreigners, and  $P_t$  denotes the remaining items, consisting of the stocks of portfolio equity, financial derivatives, and international reserves. It then follows that  $S_t - S_{t-1} = P_t - P_{t-1} - FDI_t$ , where  $FDI_t$  is the net *flow* of foreign direct investment. In addition, letting  $V_t = P_t - P_{t-1} - \omega_t$ , the last equation becomes

$$D_t = D_{t-1} (1 + r_t) - NICA_t - FDI_t + V_t. \quad (\text{A.2})$$

That is, the stock of foreign debt at time  $t$  equals the previous stock plus interest payments minus the non-interest current account, minus the net flow of foreign direct investment plus



the term  $V_t$  which includes the changes in portfolio equity, financial derivatives, international reserves, valuation changes, and debt relief. The latter enters with a negative sign and is an important item for low income countries.

Let  $Q_t$  denote GDP measured in U.S. dollars and write (A.2) as

$$d_t = d_{t-1} \left( \frac{1 + r_t}{Q_t/Q_{t-1}} \right) - m_t - f_t + v_t. \quad (\text{A.3})$$

where  $d_t = D_t/Q_t$ ,  $m_t = NICA_t/Q_t$ ,  $f_t = FDI_t/Q_t$ , and  $v_t = V_t/Q_t$ . Note that  $Q_t/Q_{t-1}$  is the (gross) growth rate of GDP measured in U.S. dollars. The objective now is to write  $Q_t/Q_{t-1}$  in terms of the growth rate of real GDP (in constant local currency units) and the growth rate of the GDP deflator measured in U.S. dollars (a term proportional to the reciprocal of the real exchange rate). Let  $\mathcal{E}_t$  denote the nominal exchange rate of local currency per U.S. dollar,  $\mathcal{P}_t$  the GDP deflator in local currency, and  $q_t$  real GDP measured at constant local currency units. Then  $q_t = Q_t/(\mathcal{P}_t/\mathcal{E}_t)$  and thus,

$$\begin{aligned} \frac{Q_t}{Q_{t-1}} &= \frac{q_t}{q_{t-1}} \frac{\mathcal{P}_t/\mathcal{E}_t}{\mathcal{P}_{t-1}/\mathcal{E}_{t-1}} \\ &= (1 + g_t)(1 + \pi_t), \end{aligned}$$

where  $g_t$  is the growth rate of real GDP and  $\pi_t$  is the growth rate of the GDP deflator measured in U.S. dollars. Introducing this expression into (A.3) gives equation (2.1).

**TABLE 1**  
**Countries and number of observations**

Country	Years	Country	Years	Country	Years
Albania	1993-2007	Guinea	1988-2007	Paraguay	1971-1976
Angola	1990-2007	Guinea-Bissau	2000-2007	Rwanda	1971-2007
Armenia	1994-2007	Guyana	1971-2007	Samoa	2004-2007
Bangladesh	1984-2007	Haiti	1995-2007	Sao Tome and Principe	2002-2007
Benin	1988-2007	Honduras	1971-2007	Senegal	1971-2007
Bhutan	2002-2007	Jordan	1976-1977	Sierra Leone	1971-2007
Bolivia	1971-2007	Kenya	1975-2007	Solomon Islands	1991-2007
Burkina Faso	1973-2007	Kyrgyz Rep.	1995-2007	Sri Lanka	1977-2007
Burundi	1971-2007	Lao PDR	1988-2007	St. Kitts and Nevis	1985-1993
Cambodia	1994-2007	Lesotho	1980-2007	St. Lucia	1982-2007
Cameroon	1971-2007	Liberia	1979-1986	St. Vincent and the Gren.	1971-2007
Cape Verde	1987-2007	Macedonia, FYR	1995-2001	Sudan	1996-2007
Central African Rep.	1974-2007	Madagascar	1971-2007	Tajikistan	1993-2007
Chad	1971-2007	Malawi	1996-2007	Tanzania	1990-2007
Comoros	1987-2007	Maldives	1996-2007	Thailand	1971-1978
Congo, Dem. Rep.	1998-2007	Mali	1971-2007	Togo	1971-2007
Congo, Rep.	1971-2007	Mauritania	1977-2007	Tonga	2003-2007
Cote d'Ivoire	1971-2007	Moldova	1996-2007	Tunisia	1971-1976
Djibouti	1991-2007	Mongolia	1993-2007	Uganda	1991-2007
Dominica	1982-2007	Mozambique	1986-2007	Vanuatu	1982-2007
Egypt, Arab Rep.	1975-1998	Myanmar	1989-2004	Vietnam	1991-2007
Eritrea	1997-2007	Nepal	1996-2007	Yemen, Rep.	1991-2007
Ethiopia	1997-2007	Nicaragua	1995-2007	Zambia	1972-2007
Georgia	1997-2007	Niger	1971-2007	Zimbabwe	1971-2005
Ghana	1971-2007	Nigeria	1990-2007		
Grenada	1982-2007	Papua New Guinea	1971-2007		

This table reports the list low income countries included in the database and the years for which there is complete data for all of the endogenous variables.

**TABLE 2**  
**Variables and sources**

Variable	Definition	Original Source
GDP growth	Log difference of real GDP	WDI 1/
Growth of GDP deflator in US dollars	Log difference U.S. dollar value of the GDP deflator	WDI
Implicit interest rate 3/	Interest on total external debt over total debt stock	WDI
NICA/GDP 4/	Non-interest current account balance to GDP ratio	WEO 2/
FDI/GDP	Net Foreign Direct Investment to GDP ratio	WEO
Debt shocks/GDP	Derived from debt accumulation equation to GDP ratio	WDI/WEO
Exports/GDP growth	Log difference of Exports to GDP ratio	WDI
Terms of trade	Log terms of trade index (level = 100 in 2000)	WDI
World Log GDP difference	Log difference of World Real GDP	WDI
US treasury rate	Yield on U.S. Treasury securities at 1 year constant maturity	U.S. Federal Reserve System
Log Oil Price	Log Dubai crude oil price	WDI

1/ World Development Indicators

2/ World Economic Outlook

3/ Current-year interest payments on external debt divided by previous period external debt stock.

4/ NICA is Current account balance plus interest payments on external debt

The data consist of 76 low income countries. If a country moves from low income to middle income, only the low income part of the sample is kept. Two outliers were deleted from the sample: Liberia in 2007-2008 due to a huge drop in the current account balance, and Guinea in 1986, due to a unusual movement in the GDP deflator in U.S. dollars. WDI data on non-interest current account and FDI had too many missing observations. For that reason, WEO data was used instead. However, if several years of these variables are missing, data from WDI is used when available. If just one observation was missing, the observations were interpolated using the two adjacent values.

**TABLE 3**  
**Summary statistics**

	Mean	Median	Std Dev	Max	Min
GDP growth	0.036	0.040	0.054	0.301	-0.697
Growth of GDP deflator in US dollars	0.038	0.041	0.149	1.136	-1.324
Implicit interest rate	0.026	0.020	0.022	0.243	0.000
NICA/GDP	-0.049	-0.043	0.096	0.462	-0.942
FDI/GDP	0.028	0.012	0.057	0.487	-0.469
Debt shocks/GDP	-0.004	0.001	0.136	0.804	-1.554
Log terms of trade	4.680	4.627	0.340	6.273	2.054
World GDP growth	0.031	0.033	0.011	0.063	0.003
US treasury rate	0.058	0.055	0.028	0.148	0.012
Log price of oil	3.000	2.920	0.676	4.225	0.525

This table reports summary statistics of data pooled across countries

**TABLE 4**  
**Pairwise correlations (percent)**

	GDP	Deflator	Int. rate	NICA	FDI	Debt shock	TOT	World	Treas.	Oil
GDP growth	100									
Growth of GDP deflator in US dollars	-4.2	100								
Implicit interest rate	-1.9	4.0	100							
NICA/GDP	2.0	5.5	11.9	100						
FDI/GDP	9.6	1.8	-3.7	-45.2	100					
Debt shock/GDP	1.1	10.8	17.6	45.9	2.0	100				
Log terms of trade	0.2	3.9	17.9	17.6	-15.1	11.8	100			
World GDP growth	8.7	8.7	-0.5	7.1	-2.0	-0.2	7.2	100		
US treasury rate	-9.8	-4.6	46.4	2.3	-17.4	0.3	22.4	1.1	100	
Log price of oil	8.1	0.4	-10.5	-11.5	22.6	-9.0	-12.6	-16.6	-4.3	100

This table reports contemporaneous pairwise correlations in percentage points pooling observations across countries and time periods

**TABLE 5**  
**Lag structure selection**

q \ p	1		2	
	SIC	AIC	SIC	AIC
0	<b>-18.75</b>	-18.97	-18.73	-19.08
1	-18.72	-19.02	-18.69	-19.13
2	-18.67	-19.06	-18.65	<b>-19.17</b>

This table presents information criteria to choose the order of the baseline Panel VARX. The letter "p" indicates the number of lags on endogenous variables, the letter "q" indicates the number of lags on exogenous variables. SIC stands for the Schwarz Information Criteria and AIC, for the Akaike Information Criteria. Bold figures indicate the minimum SIC/AIC.

**TABLE 6**  
**Normality, excess kurtosis, and skewness tests**

Residuals	Normality	Kurtosis	Skewness
GDP Growth	63646 ***	62728 ***	918 ***
Growth of GDP deflator in US dollars	12813 ***	12258 ***	556 ***
Implicit interest rate	240789 ***	235832 ***	4957 ***
NICA/GDP	11816 ***	11671 ***	146 ***
FDI/GDP	32144 ***	31659 ***	484 ***
Debt shock/GDP	21612 ***	20897 ***	715 ***
Joint Test	389841 ***	381945 ***	7897 ***

This table shows tests of normality, skewness, and kurtosis based on Urzua (1996). Individual residuals are tested except in the last row which is a joint test. Univariate normality tests is distributed chi-square with 2 d.f. under the null of normality. Joint normality test is distributed chi-square with 12 d.f. under the null of normality. Univariate skewness and kurtosis tests are distributed chi-square with 1 d.f. under the null of zero skewness and zero kurtosis. Joint test of skewness and kurtosis are distributed chi-square with 6 d.f. under the null of zero skewness and zero kurtosis. Note: Asteriks (\*\*\*) indicate rejection of the null at 1% significance level.

**TABLE 7**  
**Estimated coefficients and residuals**

	Coefficient on lagged values	Coefficients on exogenous variables	Residuals std. dev. (diagonal) and correlation
<i>LSDV estimator</i>			
$\Theta_1 =$	$\begin{bmatrix} 0.13 & -0.01 & -0.03 & 0.04 & 0.11 & -0.01 \\ 0.26 & 0.12 & -0.16 & -0.05 & 0.09 & 0.11 \\ 0.02 & 0.00 & 0.52 & 0.01 & 0.01 & -0.01 \\ -0.06 & -0.03 & 0.10 & 0.54 & -0.20 & 0.00 \\ 0.04 & 0.00 & -0.04 & -0.04 & 0.61 & -0.01 \\ -0.11 & -0.03 & 0.43 & 0.30 & 0.16 & 0.05 \end{bmatrix}$	$\Psi_0 = \begin{bmatrix} 0.01 & 0.41 & -0.10 & 0.01 \\ 0.02 & 1.05 & -0.26 & 0.00 \\ 0.01 & -0.04 & 0.16 & 0.00 \\ 0.04 & 0.54 & -0.14 & 0.00 \\ 0.00 & 0.09 & -0.12 & 0.01 \\ 0.07 & -0.02 & -0.24 & -0.01 \end{bmatrix}$	$\begin{bmatrix} \mathbf{5.0} & -7.1 & 5.5 & 3.4 & -0.3 & 0.9 \\ -7.1 & \mathbf{13.8} & 6.6 & 8.7 & -3.9 & 9.7 \\ 5.5 & 6.6 & \mathbf{1.2} & 1.9 & 1.3 & 15.0 \\ 3.4 & 8.7 & 1.9 & \mathbf{6.0} & -33.4 & 32.1 \\ -0.3 & -3.9 & 1.3 & -33.4 & \mathbf{3.5} & 8.1 \\ 0.9 & 9.7 & 15.0 & 32.1 & 8.1 & \mathbf{11.8} \end{bmatrix}$
<i>Bias corrected estimator</i>			
$\Theta_1 =$	$\begin{bmatrix} 0.19 & 0.00 & 0.00 & 0.04 & 0.11 & -0.01 \\ 0.26 & 0.17 & -0.15 & -0.05 & 0.07 & 0.11 \\ 0.02 & 0.00 & 0.59 & 0.01 & 0.01 & -0.01 \\ -0.06 & -0.03 & 0.11 & 0.61 & -0.20 & -0.01 \\ 0.04 & 0.00 & -0.03 & -0.04 & 0.68 & -0.01 \\ -0.11 & -0.03 & 0.40 & 0.31 & 0.17 & 0.11 \end{bmatrix}$	$\Psi_0 = \begin{bmatrix} 0.01 & 0.41 & -0.10 & 0.01 \\ 0.02 & 1.02 & -0.26 & 0.00 \\ 0.01 & -0.03 & 0.14 & 0.00 \\ 0.03 & 0.54 & -0.15 & 0.00 \\ 0.00 & 0.08 & -0.10 & 0.01 \\ 0.06 & -0.03 & -0.23 & -0.01 \end{bmatrix}$	$\begin{bmatrix} \mathbf{5.0} & -7.1 & 5.6 & 3.4 & -0.3 & 0.9 \\ -7.1 & \mathbf{13.9} & 6.6 & 8.7 & -3.9 & 9.7 \\ 5.6 & 6.6 & \mathbf{1.2} & 1.9 & 1.3 & 14.9 \\ 3.4 & 8.7 & 1.9 & \mathbf{6.0} & -33.5 & 32.1 \\ -0.3 & -3.9 & 1.3 & -33.5 & \mathbf{3.5} & 8.0 \\ 0.9 & 9.7 & 14.9 & 32.1 & 8.0 & \mathbf{11.8} \end{bmatrix}$

This table report estimated coefficients using a least squares dummy variable estimator (top panel) and the bootstrap based bias corrected estimator (lower panel). The diagonals of the "Residual std. dev." matrix display the standard deviation of estimated residuals in percentage points; the off-diagonals display the correlation coefficient between estimated residuals in percentage points.

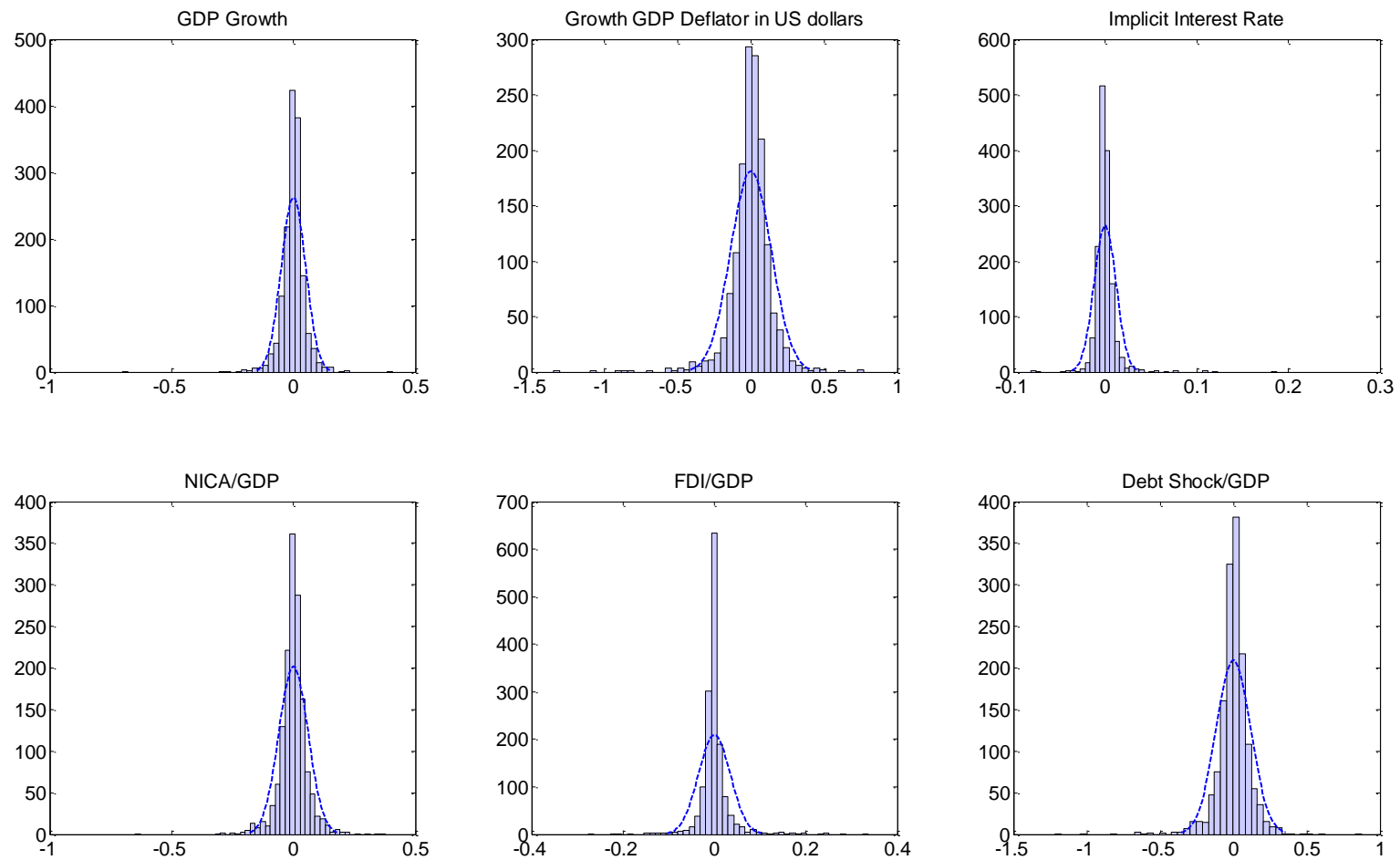
**TABLE 8****Initial conditions and long-run values for example economy**

Variable/Period	-1	0	Long run
Debt/GDP	-	0.45	0.69
GDP Growth	-	0.03	0.04
Growth of GDP deflator in US dollars	-	0.05	0.01
Implicit interest rate	-	0.02	0.02
NICA/GDP	-	-0.07	-0.04
FDI/GDP	-	0.03	0.02
Debt shock/GDP	-	-0.01	0.00
(log) terms of trade	-	log(100)	log(100)
World growth	0.02	0.02	0.035
U.S. interest rate	0.04	0.04	0.06
(log) price of oil	-	log(100)	log(33.5)

This table reports initial conditions and long run values for baseline experiment of debt sustainability analysis. The long run values are used to construct the country specific intercept.



**Figure 1**  
**Estimated residuals and associated normal densities**

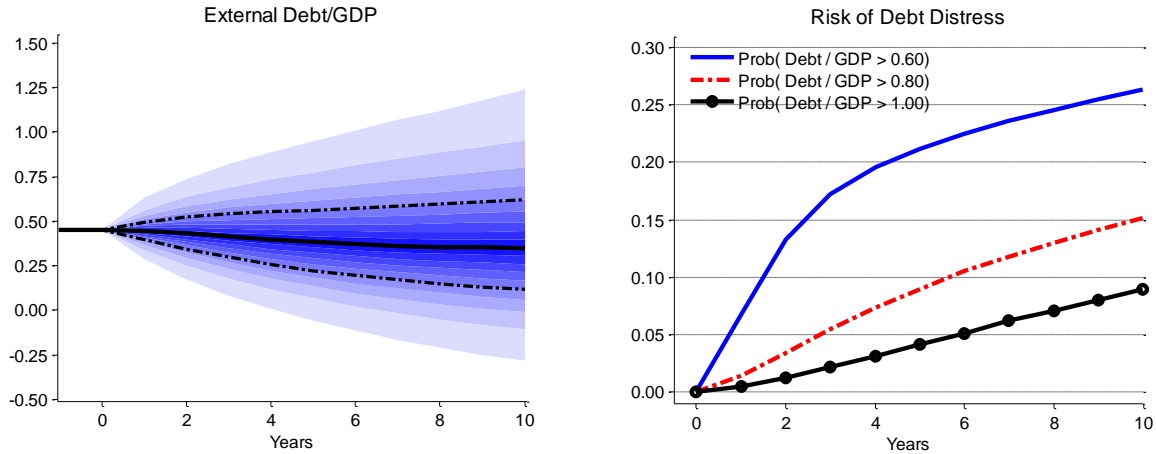


This figure shows estimated residuals from the Panel VARX estimation and the corresponding normal densities with the same mean and variance of the estimated residuals.

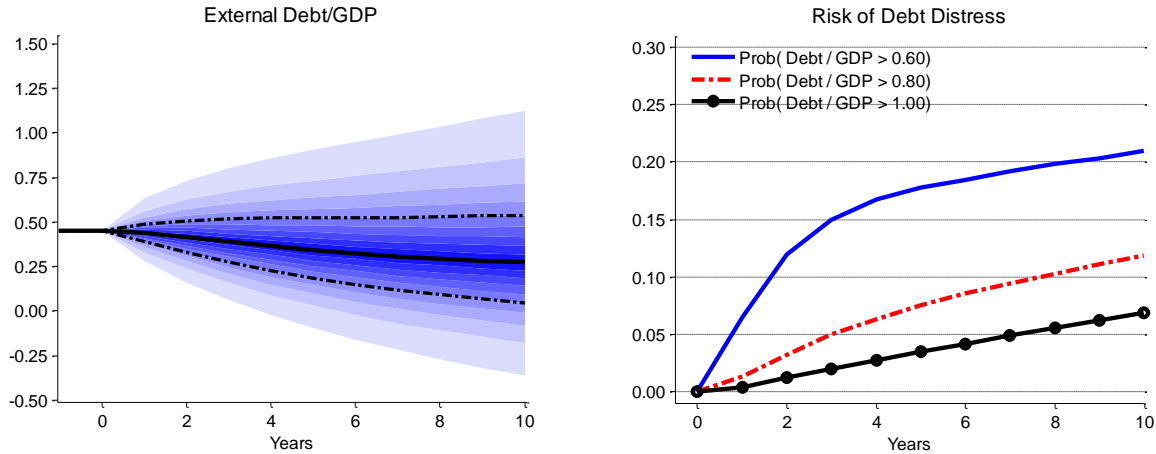
## Figure 2

Debt evolution and risk of debt distress in example economy

### Panel A. Baseline economy



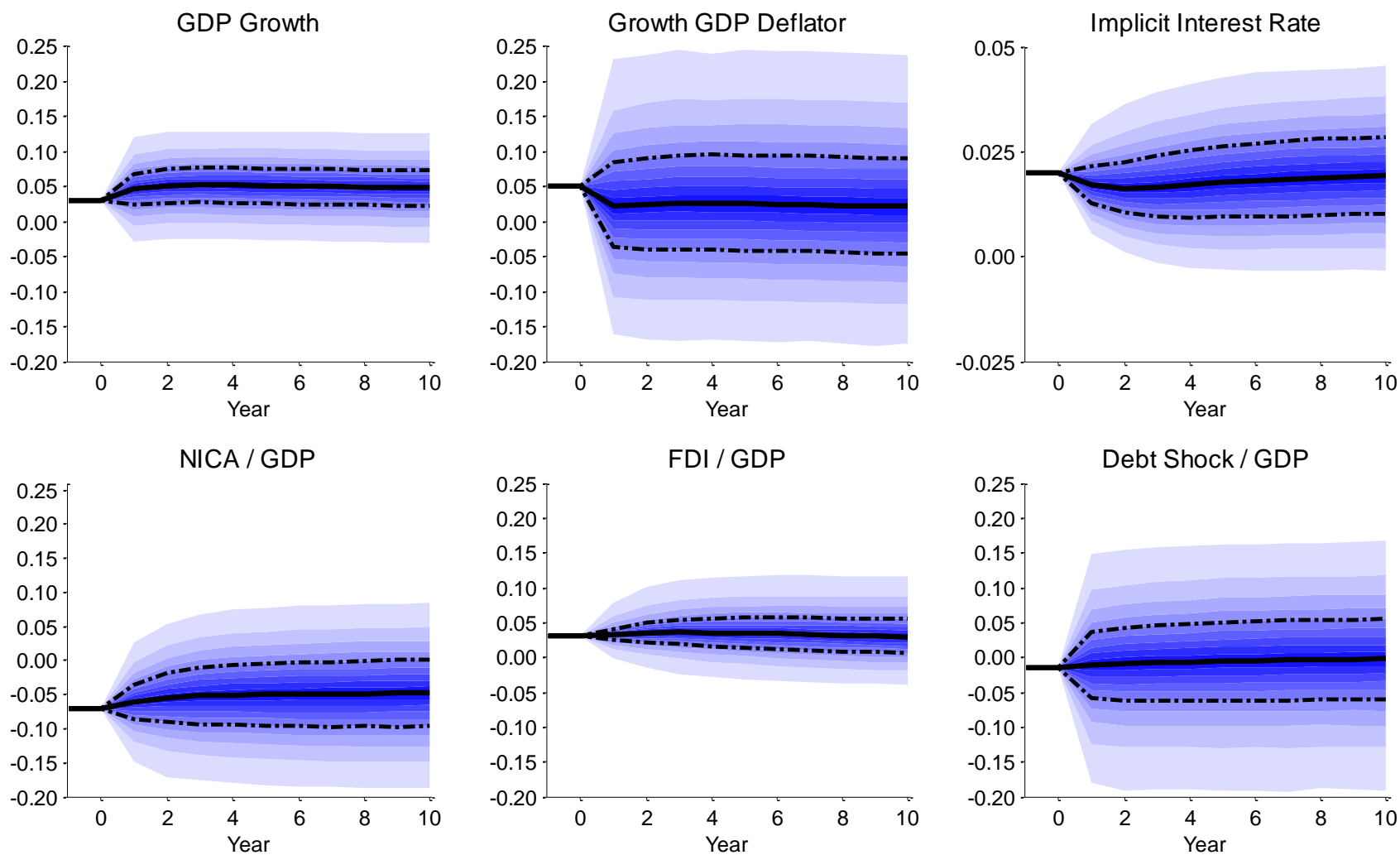
### Panel B. Reform scenario: FDI flows increase 1 percentage point of GDP in the long run



This figure shows debt trajectories, percentiles, and probability of debt crossing predetermined thresholds in the example economy. The top panel corresponds to the baseline experiment. The lower panel corresponds to a reform scenario where the flow of FDI as a fraction of GDP is increased by 1 percentage point in the long run. In the left figures the solid line is the median trajectory of foreign debt / GDP. The dashed lines are the first and third quartiles of the distribution. Shaded areas represent percentiles in 5 percent increment. The figures on the right display the probabilities, as of time zero, that the foreign debt-GDP ratio crosses a threshold of 60 (solid line), 80 (dashed line), and 100 (circled line) percentage points of GDP.

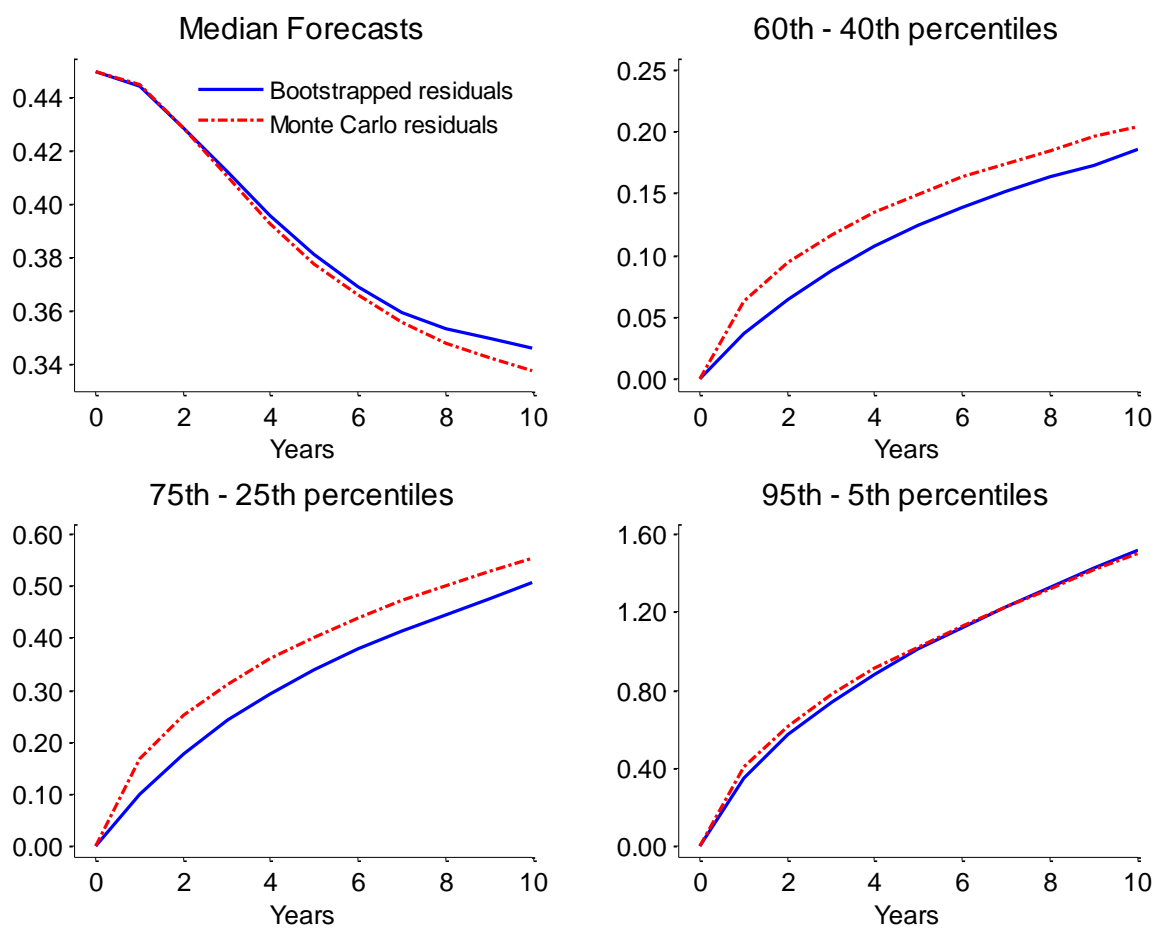
**Figure 3**

**Determinants of Foreign Debt / GDP in the example economy**



**Figure 4**

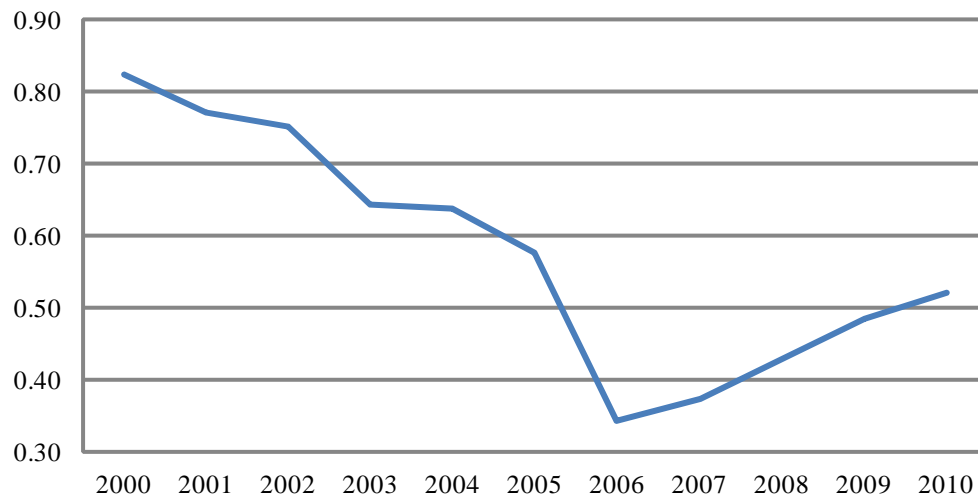
Forecast foreign debt / GDP with bootstrapped and Monte Carlo residuals



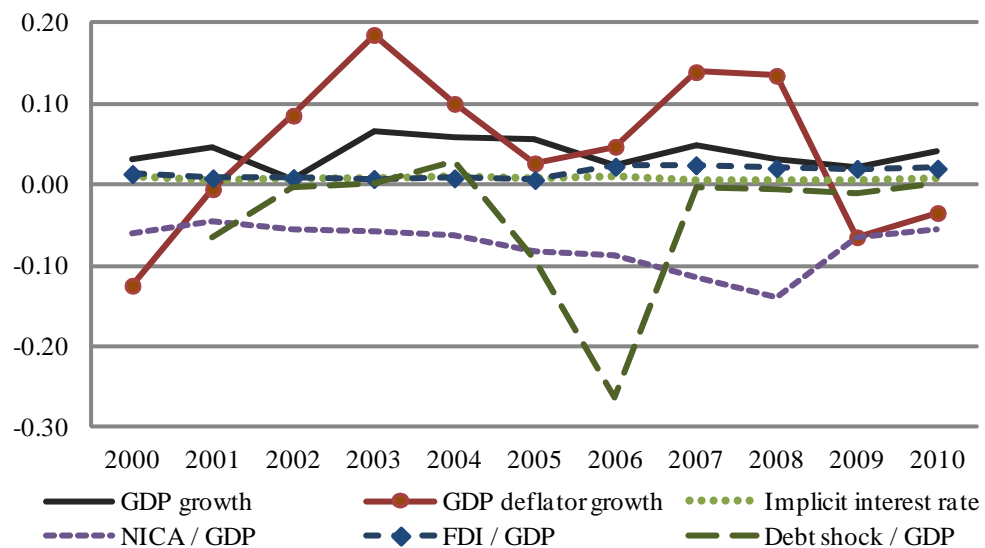
The upper left panel shows median forecasts of foreign debt / GDP in the example economy for simulation using the bootstrap method (solid line) and a Monte Carlo method (dashed line) based on sampling residuals from a normal distribution. The remaining panels show inter-percentile ranges of foreign debt/GDP ratio when drawing residuals using the bootstrap (solid line) and Monte Carlo method (dashed line)

# Figure 5

## Panel A: Evolution of foreign debt in Senegal (percentage of GDP)



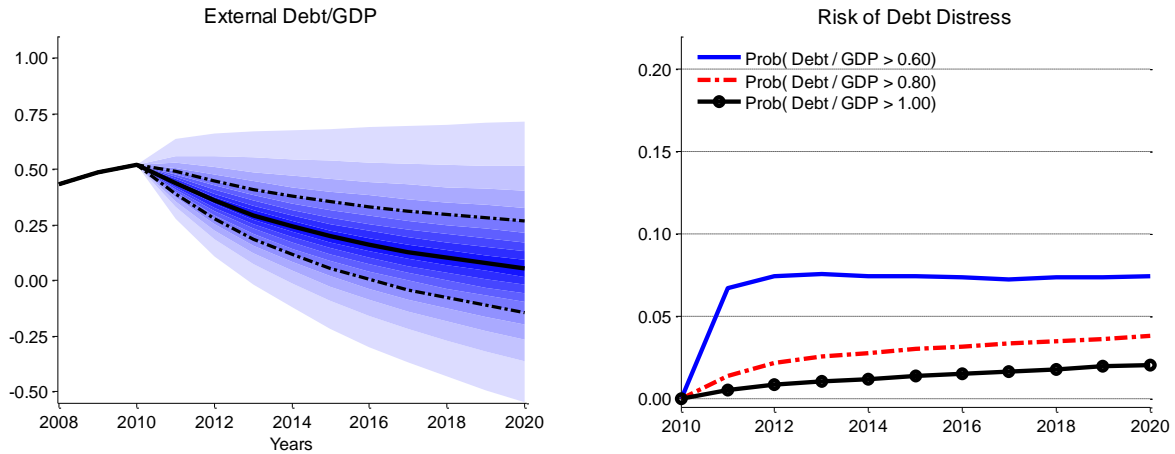
## Panel B: Evolution of the determinants of debt in Senegal



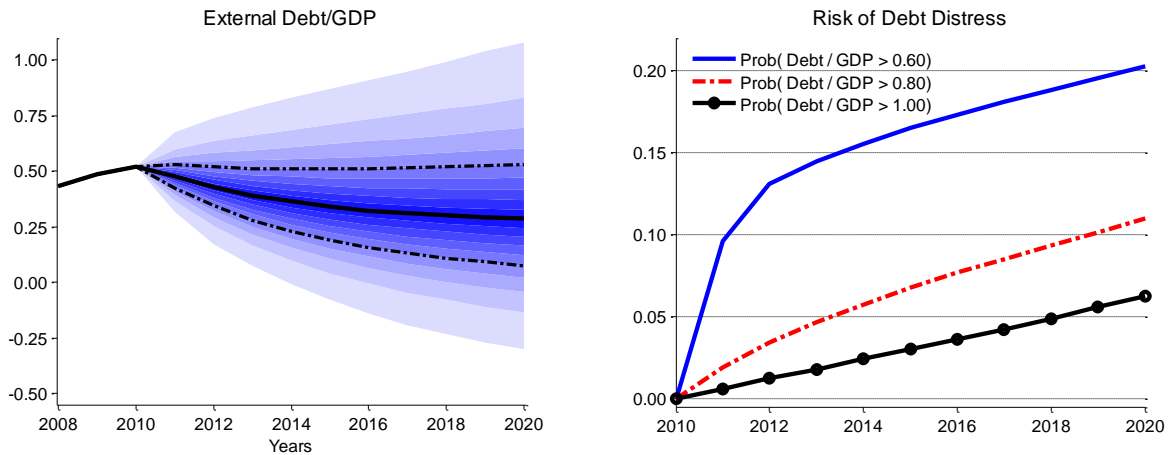
## Figure 6

### Debt evolution and risk of debt distress in Senegal

#### Panel A. Projections based on historical average in Senegal

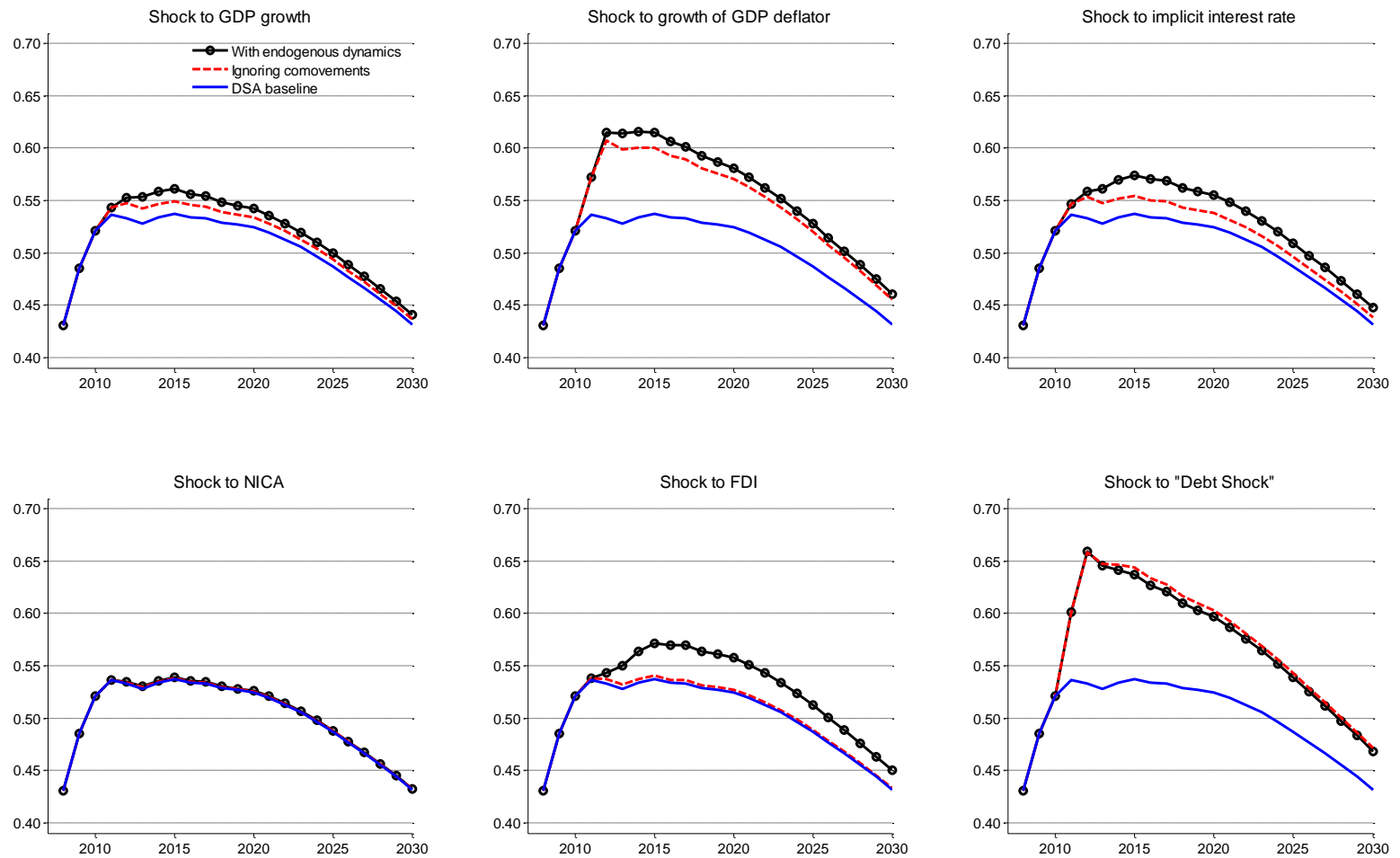


#### Panel B. Projections based on cross-country averages



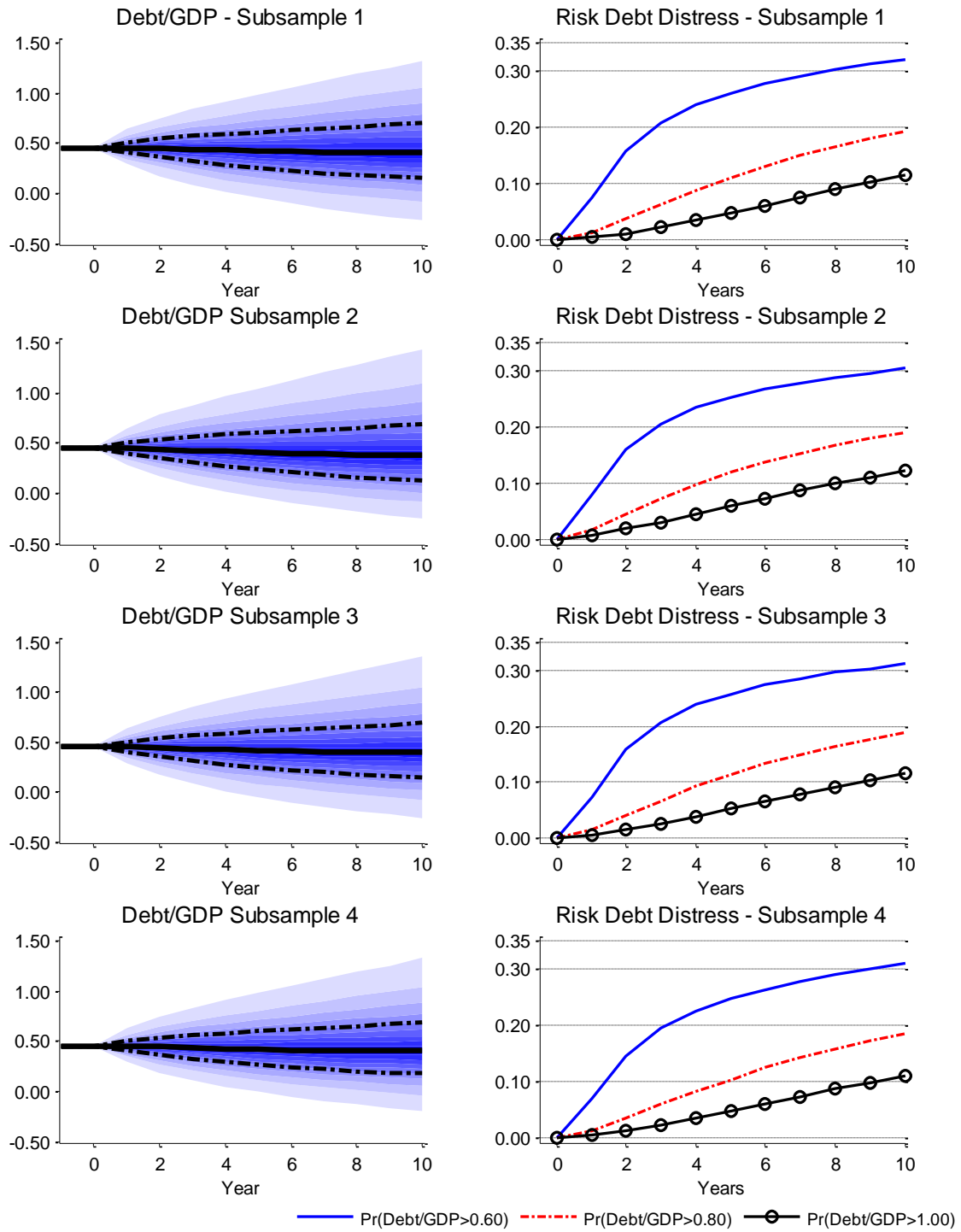
This figure shows debt trajectories, percentiles, and probability of debt crossing predetermined thresholds in the example economy. The top panel corresponds to the baseline experiment. The lower panel corresponds to a reform scenario where the flow of FDI as a fraction of GDP is increased by 1 percentage point in the long run. In the left figures the solid line is the median trajectory of foreign debt / GDP. The dashed lines are the first and third quartiles of the distribution. Shaded areas represent percentiles in 5 percent increment. The figures on the right display the probabilities, as of time zero, that the foreign debt-GDP ratio crosses a threshold of 60 (solid line), 80 (dashed line), and 100 (circled line) percentage points of GDP.

**Figure 7**



This figure displays the evolution of foreign debt based on typical World Bank - IMF Debt sustainability analysis (IDA and IMF, 2010) and six "stress tests" that consist of adding a shock of one standard deviation to each determinant of debt. The solid line is the baseline projection; the circled line is the prediction taking into account the comovements and feedback effects across determinants of debt; and the dashed line is the prediction ignoring all comovements in the determinants of debt. The difference between the circled and dashed lines can be interpreted as the error induced by ignoring comovements.

**Figure 8**

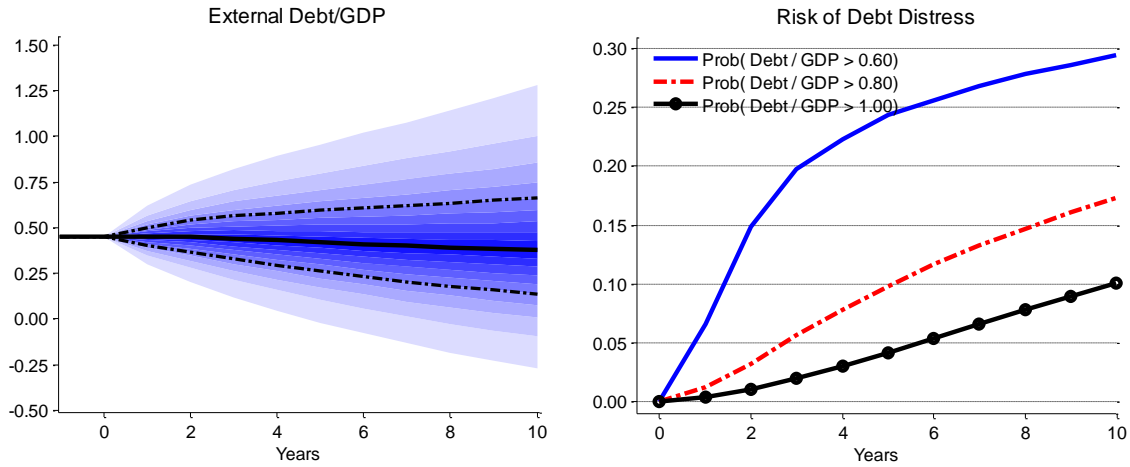


This figure displays the evolution of foreign debt and probabilities of debt reaching certain thresholds in four different subsamples of countries. Each subsample randomly deletes 20 countries from the original sample and reestimates the model.

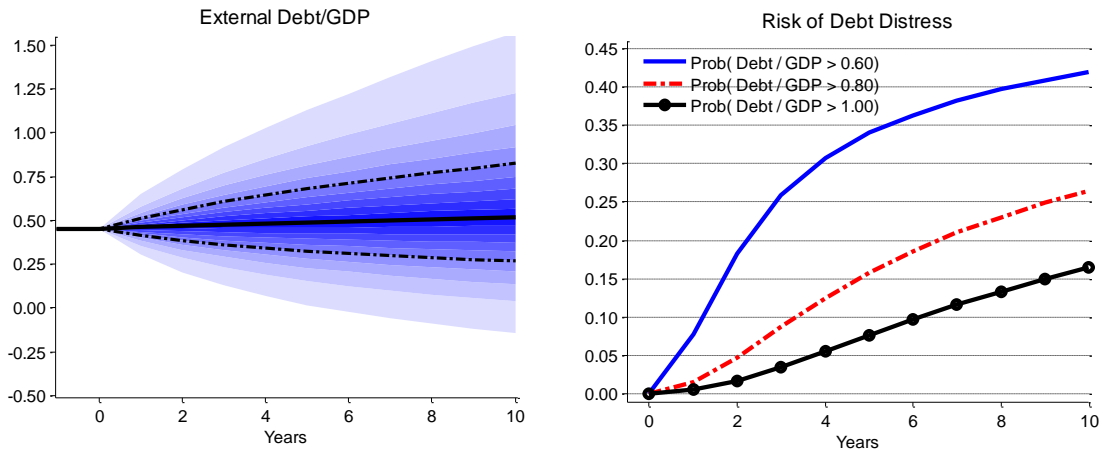


**Figure 9**  
**Predictions using different empirical models in example economy**

A. Forecasts and distress probabilities using model selected by AIC criterion



B. Forecasts and distress probabilities using model without exogenous variables



This figure displays the forecast and confidence bands of the Debt/GDP ratio and the probability of reaching debt thresholds in two alternative specifications of the empirical model. Panel A uses the model selected by the AIC criterion, with two lags in the endogenous variables and two lags in the exogenous variables. Panel B reports results for the model without exogenous variables.